REVIEW-QUESTIONS (Ch.11)

11.1. True or false: For solving a time-dependent PDE, a finite difference method that is both consistent and stable converges to the true solution as the step sizes in time and in space go to zero.

11.2. True or false: The Gauss-Seidel iterative method for solving a system of linear equations Ax = b always converges.

11.6. Other than the usual concerns of stability and accuracy, what additional important consideration enters into the choice of a numerical method for solving a system of ODEs arising from semidiscretization of a PDE using the method of lines?

REVIEW-QUESTIONS (Ch.11) (cont.)

11.7. In using a fully discrete finite difference method for solving a time-dependent PDE with one space dimension, can the sizes of the time step and space step be chosen independently of each other? Why?

11.10. Classify each of the following partial differential equations as hyperbolic, parabolic, or elliptic and state if equation is time-dependent
(a) Laplace equation
(b) Wave equation
(c) Heat equation
(d) Poisson equation

REVIEW-QUESTIONS (Ch.11) (cont.)

11.12. The heat equation $u_t = cu_{xx}$, with appropriate initial and boundary conditions can be solved numerically by using a second-order, centred finite difference approximation for u_{xx} and then solving the resulting system of ordinary differential equations in time by some numerical method.

(a) On what ODE method in time is the Crank-Nicolson method based?
(b) What advantage does the Crank-Nicolson method have over the use of the backward Euler method?
(c) What fundamental advantage do both of these methods have over the use of Euler's method?

REVIEW-QUESTIONS (Ch.11) (cont.)

11.13. In solving the Laplace equation on the unit square using the standard second-order centred finite difference scheme in both space dimensions, what is the maximum number of unknown solution variables that are involved in any one equation of the resulting linear algebraic system?

11.16. (a) For a finite difference method for solving a PDE numerically, what is meant by the terms consistency, stability, and convergence?

(b) How does the Lax Equivalence Theorem relate these terms to each other?

REVIEW-QUESTIONS (Ch.11) (cont.)

11.21. What is meant by fill in the factorization of a sparse matrix?

11.22. Suppose you factorise a matrix arising from 3x3 elliptic BVP with natural ordering of nodes: [7 8 9; 4 5 6; 1 2 3;] (lower left node is 1). Give the reordered nodes using (a) the minimum degree algorithm and (b) the nested dissection algorithm.

REVIEW-QUESTIONS (Ch.11) (cont.)

11.24. In order to implement a stationary iterative method for solving a system of linear equations Ax = b, the matrix A has to be splitted in A=M-N.

(a) For the matrix A = [4 1;1 4] what is the splitting for the Jacobi method?

(b) For the same matrix as in part d, what is the splitting for the Gauss-Seidel method?

(c) Which of the above iteration scheme will be stable?

REVIEW-QUESTIONS (Ch.11) (cont.)

11.27. Which of the following methods for solving a linear system are stationary iterative methods?

(a) Jacobi method

(b) Steepest descent method

(c) Iterative refinement

(d) Gauss-Seidel method

(e) Conjugate gradient method

(f) SOR method

11.31. What are the usual bounds on the relaxation parameter w in the SOR method?

REVIEW-QUESTIONS (Ch.11) (cont.)

11.32. Rank the following iterative methods for solving systems of linear equations in order of their usual speed of convergence, from fastest to slowest:

(a) Gauss-Seidel

(b) Jacobi

(c) SOR with optimal relaxation parameter w

REVIEW-QUESTIONS (Ch.11) (cont.)

11.34. What two key features largely account for the effectiveness of the conjugate gradient method for solving large sparse symmetric positive definite linear systems?

11.39. True or false: The V-cycle multigrid method starts with the finest grid, while the full multigrid method starts with the coarsest grid.

11.41. Is any type of method capable of solving linear systems arising from elliptic boundary value problems in time proportional to the number of grid points? If so, name one, and if not, why not?