Homework 7 – Exercise labs6_7.4

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Problem Description

Implement 20 steps of Jacobi iteration for the system matrix and boundary conditions from exercise 2. Estimate the maximal error in the iterative solution. Plot the solution.

Problem Solution

```
I = eye(4);
Z = zeros(4);
D = diag(ones(3,1),1);
D = 4*I - D - D';
D = [D -I Z Z; -I D -I Z; Z -I D -I; Z Z -I D];
b = [1 0 0 1];
b = [b b b b]';
u = D \ b;
b = [1 0 0 0 0 1];
D = [b; 1,u(13:16)',1; 1,u(9:12)',1; 1,u(5:8)',1; 1,u(1:4)',1; b];
U = [b; b; b; b; b; b];
T = U;

for n = 1:20
    for i = 2:5
        for j = 2:5
            U(i,j) = (T(i-1,j) + T(i,j-1) + T(i+1,j) + T(i,j+1)) / 4;
        end
    end
    T = U;
end

disp(sprintf('Exact solution obtained by MATLAB solver:'));
D
disp(sprintf('Approximate solution after 20 iterations:'));
U
disp(sprintf('Maximal error in the iterative solution:'));
E = D - U

figure(1); surfc(D);
figure(2); surfc(U);
figure(3); surfc(E);
```

Results

```
Exact solution obtained by MatLab solver:

D =

    1.0000         0         0         0         0    1.0000
    1.0000    0.5000    0.3333    0.3333    0.5000    1.0000
    1.0000    0.6667    0.5000    0.5000    0.6667    1.0000
    1.0000    0.6667    0.5000    0.5000    0.6667    1.0000
    1.0000    0.5000    0.3333    0.3333    0.5000    1.0000
    1.0000         0         0         0         0    1.0000

Approximate solution after 20 iterations:

U =

    1.0000         0         0         0         0    1.0000
    1.0000    0.4962    0.3272    0.3272    0.4962    1.0000
    1.0000    0.6606    0.4901    0.4901    0.6606    1.0000
    1.0000    0.6606    0.4901    0.4901    0.6606    1.0000
    1.0000    0.4962    0.3272    0.3272    0.4962    1.0000
    1.0000         0         0         0         0    1.0000

Maximal error in the iterative solution:

E =

         0         0         0         0         0         0
         0    0.0038    0.0061    0.0061    0.0038         0
         0    0.0061    0.0099    0.0099    0.0061         0
         0    0.0061    0.0099    0.0099    0.0061         0
         0    0.0038    0.0061    0.0061    0.0038         0
         0         0         0         0         0         0
```

Discussion and Comments

The above results meet the expactation that the approximate solution after 20 iterations differs only by a quantity less than 10^{-2} compared to the exact solution obtained by a MatLab solver. Due to this relatively small error, the graphical interpretations of solution matrices D and U, illustrated in Figure 1 and 2 on the next page, appear to be identical. Furthermore, error matrix E reveals the circular distribution of the maximal error in the iterative solution. As can be seen in Figure 3, this symmetry forms a conical surface.

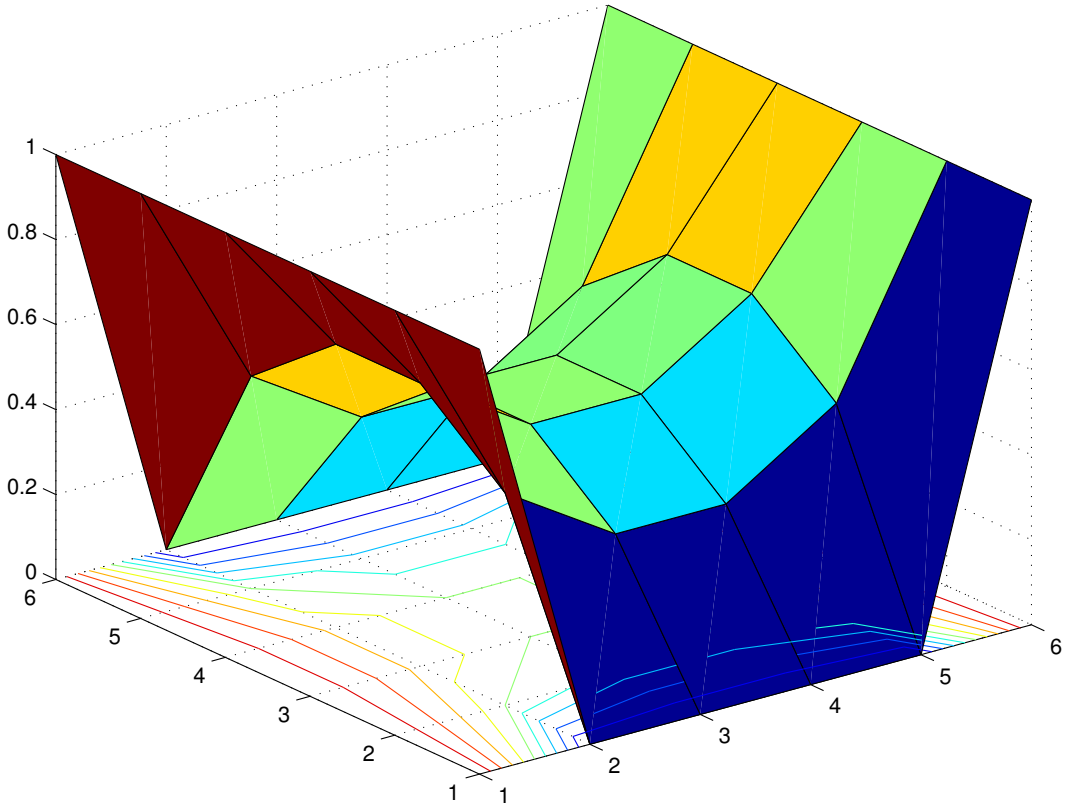

Figure 1: Exact solution obtained by MatLab solver.

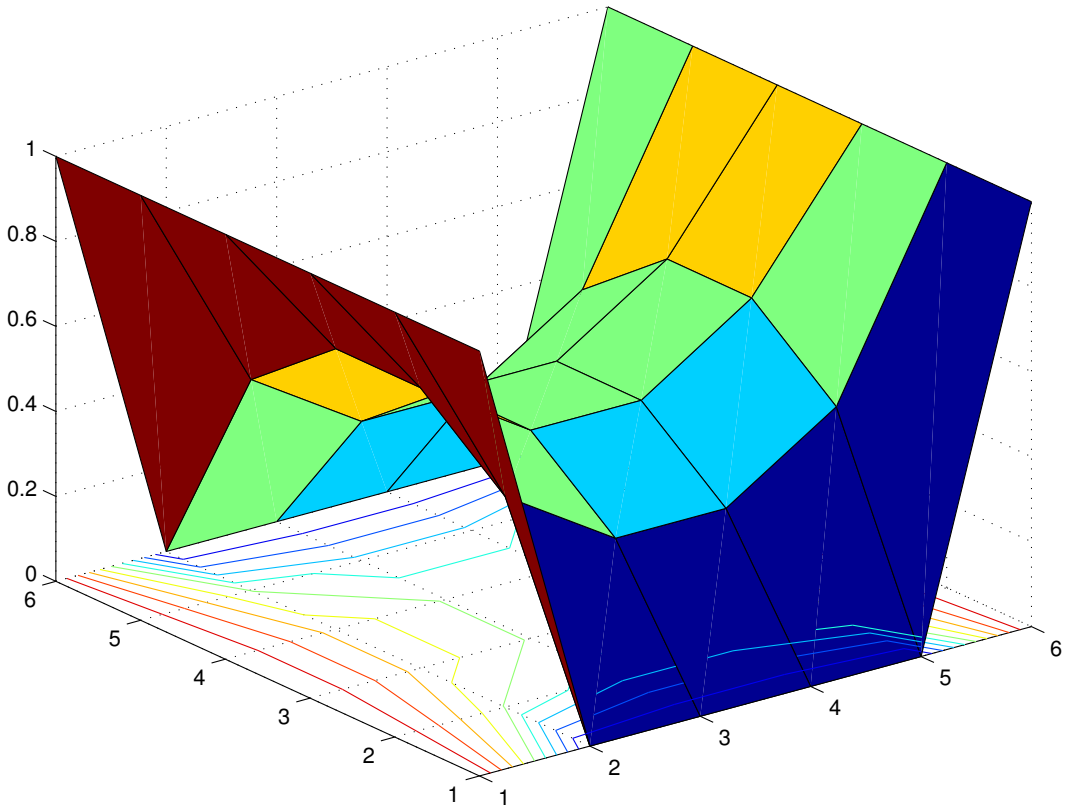

Figure 2: Approximate solution after 20 iterations.

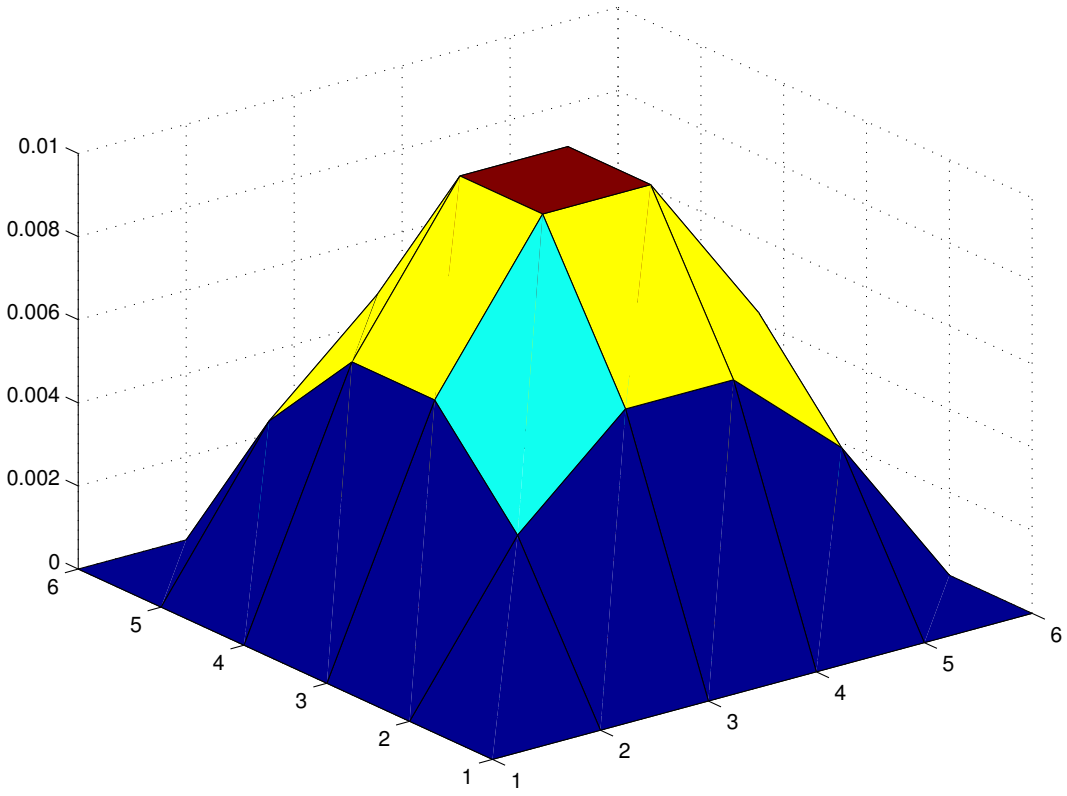

Figure 3: Maximal error in the iterative solution.