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4th Homework  
HPSC 2004

**Exercise 9.5:**

With an initial value of  $y_0 = 1$  at  $t_0 = 0$  and a time step of  $h = 1$ , compute the approximate solution value  $y_1$  at time  $t_1 = 1$  for the ODE  $y' = -y$  using each of the following two numerical methods. (Your answers should be numbers, not formulas.)

1. Euler's method
2. Backward Euler method

**My solution:**

1. **Euler's method:**

The approximate solution at time  $t_{k+1} = t_k + h_k$  is given by

$$\mathbf{y}_{k+1} = \mathbf{y}_k + h_k \cdot \mathbf{f}(t_k, \mathbf{y}_k).$$

Using this formula I get:

$$\begin{aligned} y_1 &= y_0 + h \cdot f(t_0, y_0) \\ &= 1 + 1 \cdot (-1) \\ &= 0 \end{aligned}$$

2. **Backward Euler method**

This implicit method is given by:

$$\mathbf{y}_{k+1} = \mathbf{y}_k + h_k \cdot \mathbf{f}(t_{k+1}, \mathbf{y}_{k+1}).$$

With given data I get:

$$\begin{aligned} y_1 &= y_0 + h \cdot f(t_1, y_1) \\ &= 1 + 1 \cdot (-y_1) \\ &= 1 - y_1 \\ \Rightarrow 2y_1 &= 1 \\ \Rightarrow y_1 &= 0.5 \end{aligned}$$