First name: Christian
Last name: Pfligersdorffer
Date: April 21, 2004
Homework number: 2

About the approximation of π: One of the examples in the MPI Tutorial [1] deals with a numerical approximation of π. I will go through the program code and explain what it does after the basic idea of the approximation is made clear.

The example introduces an integral that is said to equal π hence an approximation of the integral is an approximation of π. That sounds quite sensible but does the integral equal π? Let's have a look at it:

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{4}{1+x^2}\,dx = 4\arctan x\Big|_{-\frac{1}{2}}^{\frac{1}{2}} = 8\arctan\frac{1}{2}$$

The first equality holds because $\arctan x$ is an antiderivative of the integrand, the second equality is due to the symmetry of the arcus tangens: $\arctan -x = -\arctan x$. We only need to know the value of $\arctan\frac{1}{2}$ or, in other words, for what angle the tangens is one half.

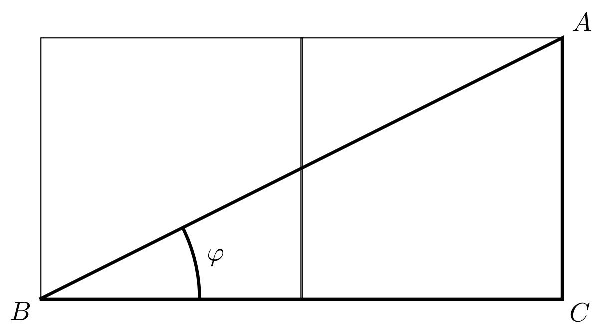

Figure 1: A specifically formed right-angled triangle.

In the right-angled triangle ΔABC shown in figure 1 the cathetus $\overline{BC}$ is twice as long as $\overline{AC}$. Applying a simple trigonometric law for the right-angled triangle, we find out, that φ is the angle we are looking for.

$$\tan\varphi = \frac{\overline{AC}}{\overline{BC}} = \frac{1}{2} \;\Leftrightarrow\; \varphi = \arctan\frac{1}{2}$$

Of course we assume the correctness of the method and that $\varphi = \frac{\pi}{8}$ but both turn out to be wrong! Figure 2 shows us how π and φ are connected.

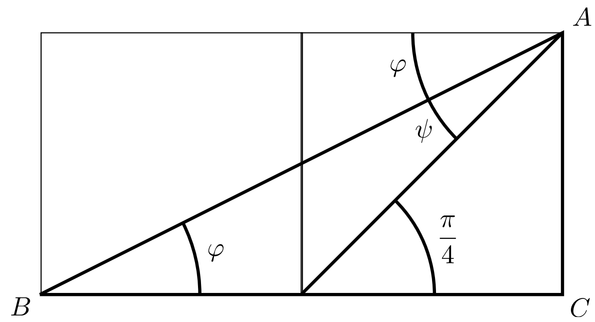

Figure 2: The two angles φ and ψ cannot be the same.

It is through the equality $\varphi + \psi = \frac{\pi}{4}$. If φ was to be $\frac{\pi}{8}$ then surely ψ would have to have the same size which is oviously not true.

All summed up the approximation method for π cannot work in the proposed way, since the value of the approximated integral is not π but larger. A minor correction on the limits of the integral could solve the discrepancy—and $\arctan 1 = \frac{\pi}{4}$ so the following equation does hold:

$$\int_0^1 \frac{4}{1+x^2}\,dx = \pi$$

The MPI program: For simplicity I have inserted the explanation as comments into the code itself. The program is not truely complicated so the comments are seldom longer that a line or so.

```
#include "mpi.h"
#include <math.h>

int main(argc,argv)
int argc;
char *argv[];
{
   int done = 0, n, myid, numprocs, i;
   double PI25DT = 3.141592653589793238462643;
   double mypi, pi, h, sum, x;

   MPI_Init(&argc,&argv);
   MPI_Comm_size(MPI_COMM_WORLD,&numprocs);
   // numprocs now is the number of nodes participating
```

```
  MPI_Comm_rank(MPI_COMM_WORLD,&myid);
  // who am I? myid tells us
  while (!done)
  {
     if (myid == 0) {
        // if I am root, let's get some input from the user
        printf("Enter the number of intervals: (0 quits) ");
        scanf("%d",&n);
     }
     MPI_Bcast(&n, 1, MPI_INT, 0, MPI_COMM_WORLD);
     // let every node know the number of intervals n
     if (n == 0) break;

     h   = 1.0 / (double) n;
     // step size according to the number of intervals
     sum = 0.0;
     for (i = myid + 1; i <= n; i += numprocs) {
        // intervals are spread among the nodes
        x = h * ((double)i - 0.5);
        // stores the current x value; a wrong pair of
        // parentheses lead to correct over-all results!
        sum += 4.0 / (1.0 + x*x);
        // the intervals being small the integral can be
        // approximated by h * integrand at x
     }
     mypi = h * sum;

     MPI_Reduce(&mypi, &pi, 1, MPI_DOUBLE, MPI_SUM, 0,
                MPI_COMM_WORLD);
     // all the local results are now brought together

     if (myid == 0)
        // if I am root, let's give the answer
        printf("pi is approximately %.16f, Error is %.16f\n",
               pi, fabs(pi - PI25DT));
  }
  MPI_Finalize();
  return 0;
}
```

Why it works nonetheless: As shown before the method for approximating π is flawed but if you run the program it gives precise results for answer nonetheless. A miracle?

After some closer inspections of the above code it is clear: A small 'mistake' led to correct behaviour in spite of the wrong method! Conspirators might believe this was done on purpose but I guess it was coincidence. By virtue of the additional parentheses (you find them in the for-loop) the algorithm computes something the author surely could not have in mind, namely an approximation of the following integral:

$$\int_{-\frac{h}{2}}^{1-\frac{h}{2}} \frac{4}{1+x^2}\, dx$$

Attention should be paid to the fact that for $h \to 0$ this integral really converges towards π, so that the algorithm reaches arbitrary precision by increasing the number of intervals and thereby decreasing the step size h.

References

[1] `http://www-unix.mcs.anl.gov/mpi/tutorial/mpiexmpl/`