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Homework number: 2
Homework Title: PI (MPI Programm)

Problem:

This exercise presents a simple program to determine the value of pi. The algorithm suggested here is chosen for its simplicity. The method evaluates the integral of $\frac{4}{1+x^2}$ between $\frac{-1}{2}$ and $\frac{1}{2}$. The method is simple: the integral is approximated by a sum of n intervals; the approximation to the integral in each interval is $\frac{1}{n} \cdot \frac{4}{1+x^2}$. The master process (rank 0) asks the user for the number of intervals; the master should then broadcast this number to all of the other processes. Each process then adds up every n'th interval $(x = \frac{-1}{2} + \frac{rank}{n}, \frac{-1}{2} + \frac{rank}{n} + \frac{\text{size}}{n})$. Finally, the sums computed by each process are added together using a reduction.

--> solution next page

Solution:

```
#include "mpi.h" /* provides basic MPI definitions and types */
#include <math.h> /* provides basic mathematical functions*/

int main(argc,argv)
int argc;
char *argv[];
{
    int done = 0, n, myid, numprocs, i;
    /*true value of pi (rounded) for comparision*/
    double PI25DT = 3.141592653589793238462643;
    /*instanz variable*/
    double mypi, pi, h, sum, x;

    /* starts MPI*/
    MPI_Init(&argc,&argv);
    /*the number of all processors are written into the variable numprocs*/
    MPI_Comm_size(MPI_COMM_WORLD,&numprocs);
    /* determines the processor id for each processor (from 0 to size-1)*/
    MPI_Comm_rank(MPI_COMM_WORLD,&myid);

    while (!done)
    {
      /* the process with the id 0 starts and asks the user for input*/
      if (myid == 0) {
          printf("Enter the number of intervals: (0 quits) ");
          scanf("%d",&n);
      }
      /* the process with the id 0 sends the same data to all other prozesses,
      the data is a single MPI_INT variable namely n*/
      MPI_Bcast(&n, 1, MPI_INT, 0, MPI_COMM_WORLD);
      if (n == 0) break;
      /* determines the length of a subintervall*/
      h   = 1.0 / (double) n;
      sum = 0.0;
      /* the main part of the programm where the integral for all x is computed*/
      for (i = myid + 1; i <= n; i += numprocs) {
          /*current interior point*/
          x = h * ((double)i - 0.5);
          /* each result of the subintervall approximation is added*/
          sum += 4.0 / (1.0 + x*x);
      }
      /* mypi is a part of pi computed by the current process*/
      mypi = h * sum;
      /* combines mypi from all processes using MPI_SUM*/
      MPI_Reduce(&mypi, &pi, 1, MPI_DOUBLE, MPI_SUM, 0,
               MPI_COMM_WORLD);
      /*the root process (with the id 0) compares the computed result with the true
      value of π*/
      if (myid == 0)
          printf("pi is approximately %.16f, Error is %.16f\n",
               pi, fabs(pi - PI25DT));
    }
    /* Ends MPI */
    MPI_Finalize();
    return 0;
}
```

Results:

We will change the problem discribed above a little because $\int_a^b \frac{4}{1+x^2} = 4\int_a^b \frac{1}{1+x^2} = 4(\arctan b - \arctan a)$ should be equal to π..

The boundarys $\frac{-1}{2}$ and $\frac{1}{2}$ don`t fit the requirements because

$$\int_{-0.5}^{0.5} \frac{4}{1+x^2} = 4(\arctan 1 - \arctan 0) \neq \pi .$$

We take the boundarys 0 and 1 which give an accurate result:

$$\int_0^1 \frac{4}{1+x^2} = 4(\arctan 1 - \arctan 0) = 4(\frac{\pi}{4} + 0) = \pi .$$

So it is easier to descripe the interprocess communication.

Interprocess communication:

To illustrate how the interprocess communication works we will descripe it with reference to a division of [0,1] in 4 subintervall ($n = 4$) $[0, \frac{1}{4}], [\frac{1}{4}, \frac{2}{4}], [\frac{2}{4}, \frac{3}{4}], [\frac{3}{4}, 1]$ and compute the interior point as following $(x = h * ((double) i))$. We assume that the number of prozesses is 3.

So the first (root, id = 0) prozess approximates the first and last intervall

$$mypi = h \cdot \text{sum} = (\frac{1}{4}) \cdot \frac{4}{1+(\frac{1}{4})^2} = \frac{1}{1+\frac{1}{16}} = \frac{16}{17}, \quad \frac{1}{4} \cdot \frac{4}{1+1^2} = \frac{1}{2}$$

the second prozess (id = 1) approximates the second intervall $mypi = \frac{1}{4} \cdot \frac{4}{1+(\frac{2}{4})^2} = \frac{1}{1+\frac{4}{16}} = \frac{4}{5}$ and the third prozess (id = 2) the third intervall $mypi = \frac{1}{4} \cdot \frac{4}{1+(\frac{3}{4})^2} = \frac{1}{1+\frac{9}{16}} = \frac{16}{25}$.

In this case pi is $\frac{16}{17} + \frac{1}{2} + \frac{4}{5} + \frac{16}{25} = \frac{2449}{850} = 2{,}881176470$ which is 91,7 % correct.

The tutorial computed the interior point with x = h * ((double)i – 0.5) contrary to our example above where we took x = h * ((double) i). We choose x = h * ((double) i) for the illustration because the solution it produces converges much faster to π

Taking x = h * ((double)i – 0.5) and n = 4 we get: $\frac{4}{5} + \frac{4}{13} + \frac{4}{29} + \frac{4}{53} = 1.32109504029$

which is only 42,05% correct..