Homework 1 – Exercise 4

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Problem Description

Derive expressions that indicate when a 2-D decomposition of a finite difference computation on an $N \times N \times Z$ grid will be superior to a 1-D decomposition and when a 3-D decomposition will be superior to a 2-D decomposition. Are these conditions likely to apply in practice? Let $t_s = 1\,\mu\text{sec}$, $t_w = 0.04\,\mu\text{sec}$, $t_c = 1\,\mu\text{sec}$, and $P = 1000$. For what values of N does the use of a 3-D decomposition rather than a 2-D decomposition reduce execution time by more than 10 percent?

Problem Solution

Computation Time

$N \times N \times Z$ grid $\Rightarrow$ $T_{\text{comp}} = t_c N^2 Z$

1-D Decomposition

$2NZ$ channels, $4NZ$ messages $\Rightarrow$ $T_{\text{comm}} = t_s 2P + t_w 4NZP$

$$T_{\text{total 1-D}} = \frac{T_{\text{comp}} + T_{\text{comm}}}{P} = \frac{t_c N^2 Z + t_s 2P + t_w 4NZP}{P}$$

$$E_{\text{relative 1-D}} = \frac{T_1}{P T_P} = \frac{t_c N^2 Z}{t_c N^2 Z + t_s 2P + t_w 4NZP}$$

$$T_{\text{execution 1-D}} = T_{\text{total 1-D}} \times (1 - E_{\text{relative 1-D}}) = \frac{t_s 2P + t_w 4NZP}{P}$$

2-D Decomposition

$4NZ$ channels, $8\frac{N}{\sqrt{P}}Z$ messages $\Rightarrow$ $T_{\text{comm}} = t_s 4P + t_w 8NZ\sqrt{P}$

$$T_{\text{total 2-D}} = \frac{T_{\text{comp}} + T_{\text{comm}}}{P} = \frac{t_c N^2 Z + t_s 4P + t_w 8NZ\sqrt{P}}{P}$$

$$E_{\text{relative 2-D}} = \frac{T_1}{PT_P} = \frac{t_c N^2 Z}{t_c N^2 Z + t_s 4P + t_w 8NZ\sqrt{P}}$$

$$T_{\text{execution 2-D}} = T_{\text{total 2-D}} \times (1 - E_{\text{relative 2-D}}) = \frac{t_s 4P + t_w 8NZ\sqrt{P}}{P}$$

3-D Decomposition

$$6NZ \text{ channels, } 12\frac{N}{\sqrt[3]{P}}\frac{Z}{\sqrt[3]{P}} \text{ messages} \quad \Rightarrow \quad T_{\text{comm}} = t_s 6P + t_w 12NZ\sqrt[3]{P}$$

$$T_{\text{total 3-D}} = \frac{T_{\text{comp}} + T_{\text{comm}}}{P} = \frac{t_c N^2 Z + t_s 6P + t_w 12NZ\sqrt[3]{P}}{P}$$

$$E_{\text{relative 3-D}} = \frac{T_1}{PT_P} = \frac{t_c N^2 Z}{t_c N^2 Z + t_s 6P + t_w 12NZ\sqrt[3]{P}}$$

$$T_{\text{execution 3-D}} = T_{\text{total 3-D}} \times (1 - E_{\text{relative 3-D}}) = \frac{t_s 6P + t_w 12NZ\sqrt[3]{P}}{P}$$

Derive expressions that indicate when a 2-D decomposition of a finite difference computation on an $N \times N \times Z$ grid will be superior to a 1-D decomposition and when a 3-D decomposition will be superior to a 2-D decomposition.

$$\begin{aligned} T_{\text{execution 2-D}} &= \frac{t_s 4P + t_w 8NZ\sqrt{P}}{P} = \frac{t_s 2P + t_w 4NZP}{P} = T_{\text{execution 1-D}} \\ N\left(t_w 8Z\sqrt{P} - t_w 4ZP\right) &= t_s 2P - t_s 4P \\ N &= \frac{-t_s 2P}{t_w 8Z\sqrt{P} - t_w 4ZP} \end{aligned}$$

$$\begin{aligned} T_{\text{execution 3-D}} &= \frac{t_s 6P + t_w 12NZ\sqrt[3]{P}}{P} = \frac{t_s 4P + t_w 8NZ\sqrt{P}}{P} = T_{\text{execution 2-D}} \\ N\left(t_w 12Z\sqrt[3]{P} - t_w 8Z\sqrt{P}\right) &= t_s 4P - t_s 6P \\ N &= \frac{-t_s 2P}{t_w 12Z\sqrt[3]{P} - t_w 8Z\sqrt{P}} \end{aligned}$$

Hence a 2-D decomposition will be superior to a 1-D decomposition whenever

$$N \geq \frac{t_s P}{t_w 2Z\left(P - 2\sqrt{P}\right)},$$

and a 3-D decomposition will be superior to a 2-D decomposition whenever

$$N \geq \frac{t_s P}{t_w 2Z\left(2\sqrt{P} - 3\sqrt[3]{P}\right)}.$$

Are these conditions likely to apply in practice?

No, because these conditions are based on performance models that are only idealizations of more complex phenomena.

Let $t_s = 1\,\mu sec$, $t_w = 0.04\,\mu sec$, $t_c = 1\,\mu sec$, and $P = 1000$. For what values of N does the use of a 3-D decomposition rather than a 2-D decomposition reduce execution time by more than 10 percent?

Analytical Solution

$$
\begin{aligned}
T_{\text{execution 3-D}} &= T_{\text{execution 2-D}} \times 0.9 \\
\frac{t_s 6P + t_w 12NZ\sqrt[3]{P}}{P} &= \frac{t_s(4)(0.9)P + t_w(8)(0.9)NZ\sqrt{P}}{P} \\
N\left(t_w 12Z\sqrt[3]{P} - t_w 7.2Z\sqrt{P}\right) &= t_s 3.6P - t_s 6P \\
N &= \frac{t_s 2.4P}{t_w Z\left(7.2\sqrt{P} - 12\sqrt[3]{P}\right)} \\
N &= \frac{(1)(2.4)(1000)}{(0.04)(1)\left(7.2\sqrt{1000} - 12\sqrt[3]{1000}\right)} \\
N, P \in \mathbb{N} \;\Rightarrow\; \forall\, N &\geq \lceil 557.1859 \rceil = 558
\end{aligned}
$$

Numerical Solution

```
echo off;
tc = 1; ts = 1; tw = 0.04; P = 1000; N = [1:1:1000];
Er1 = N; Er2 = N; Er3 = N;
Tt1 = N; Tt2 = N; Tt3 = N;
Te1 = N; Te2 = N; Te3 = N;
sP = P^(1/2);
cP = P^(1/3);
find_Te3 = 1;

for n = N
    % Relative Efficiency
    Er1(n) = tc*n^2/(tc*n^2 + ts*2*P + tw*4*n*P);
    Er2(n) = tc*n^2/(tc*n^2 + ts*4*P + tw*8*n*sP);
    Er3(n) = tc*n^2/(tc*n^2 + ts*6*P + tw*12*n*cP);

    % Total Time
    Tt1(n) = (tc*n^2 + ts*2*P + tw*4*n*P)/P;
    Tt2(n) = (tc*n^2 + ts*4*P + tw*8*n*sP)/P;
    Tt3(n) = (tc*n^2 + ts*6*P + tw*12*n*cP)/P;

    % Execution Time
    Te1(n) = Tt1(n)*(1 - Er1(n));
    Te2(n) = Tt2(n)*(1 - Er2(n));
    Te3(n) = Tt3(n)*(1 - Er3(n));
```

```
    % Find N where execution time of 3-D decomposition
    % is more than 10% less than 2-D decomposition
    if( find_Te3 && Te3(n) < Te2(n)*0.9 )
        Te3_10_percent_less_than_Te2_at_N = n
        find_Te3 = 0;
    end
end

figure(1); hold on;
xlabel('Problem Size N'); ylabel('Relative Efficiency');
plot(N,Er1,'b-'); plot(N,Er2,'r-'); plot(N,Er3,'g-');

figure(2); hold on;
xlabel('Problem Size N'); ylabel('Total Time');
plot(N,Tt1,'b-'); plot(N,Tt2,'r-'); plot(N,Tt3,'g-');

figure(3); hold on;
xlabel('Problem Size N'); ylabel('Execution Time');
plot(N,Te1,'b-'); plot(N,Te2,'r-'); plot(N,Te3,'g-');
```

Numerical Results

```
Te3_10_percent_less_than_Te2_at_N =

   558
```

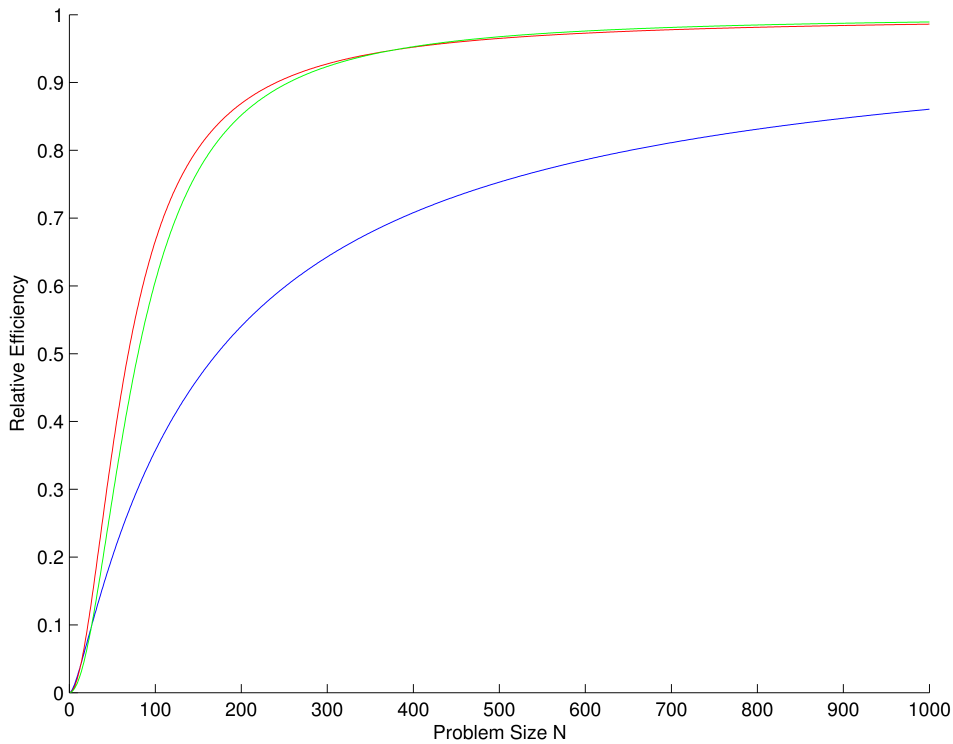

Figure 1: Relative Efficiency of 1-D (blue), 2-D (red), and 3-D (green) decomposition.

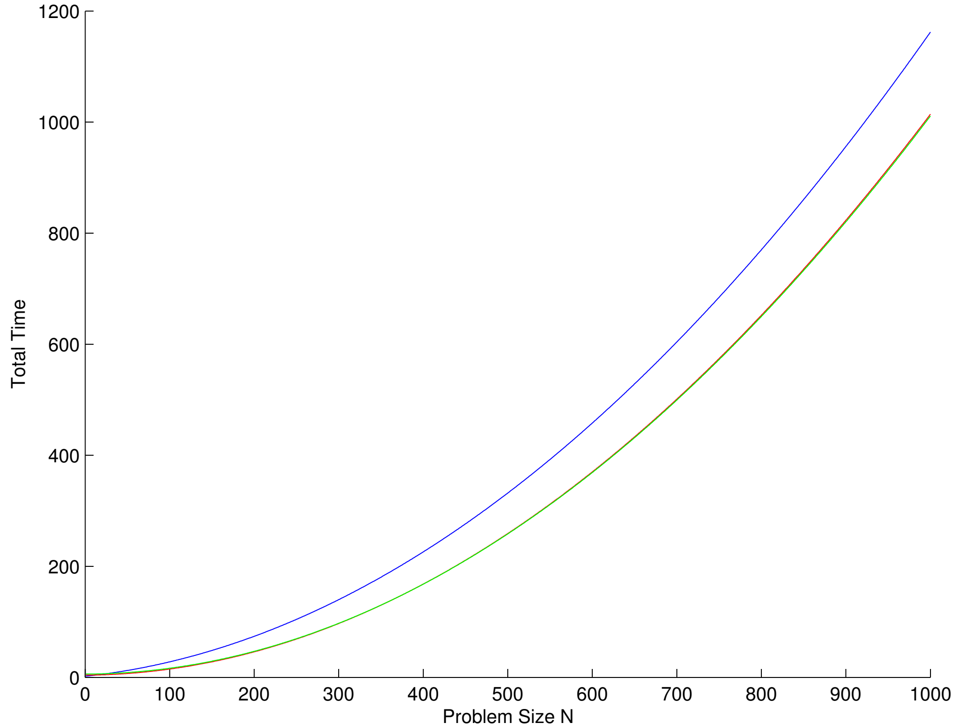

Figure 2: Total Time of 1-D (blue), 2-D (red), and 3-D (green) decomposition.

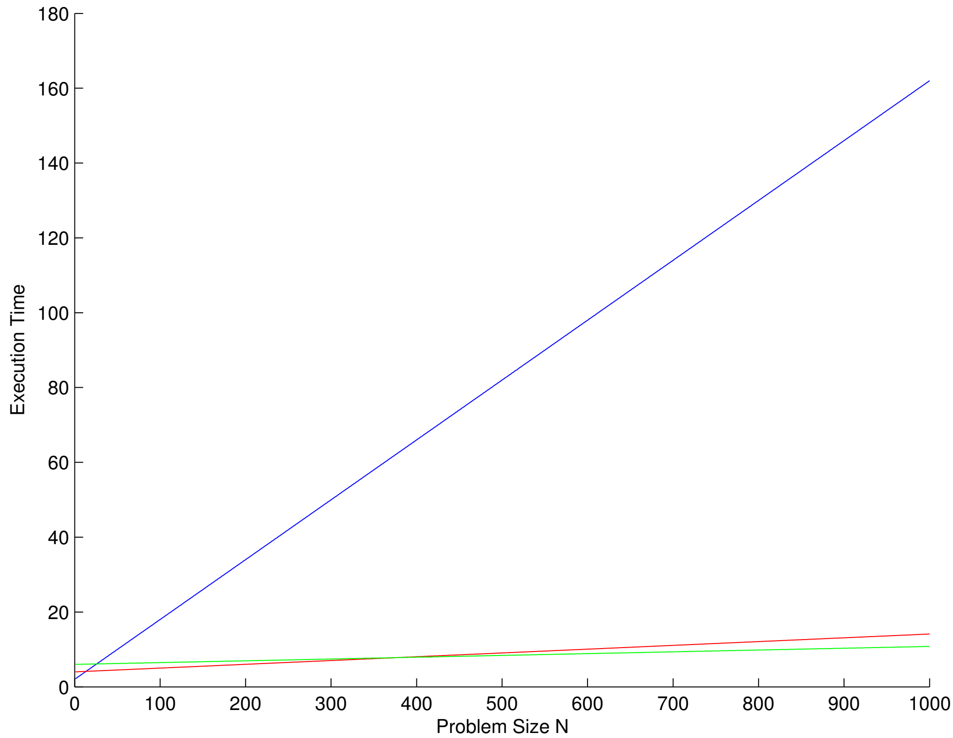

Figure 3: Execution Time of 1-D (blue), 2-D (red), and 3-D (green) decomposition.