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•8.1. True or false: Evaluating a definite integral is always a well-conditioned problem.

•8.2. True or false: Because it is based on polynomial interpolation of degree one higher, the trapezoid rule is generally more accurate than the midpoint rule.

•8.4. True or false: An *n*-point Newton-Cotes quadrature rule is always of degree *n* - 1.

•8.5. True or false: Gaussian quadrature rules of different orders never have any points in common.

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•8.9. Name two different methods for computing the weights corresponding to a given set of nodes of a quadrature rule.

•8.10. How can you estimate the error in a quadrature rule without computing the derivatives of the integrand function that would be required by a Taylor series expansion?

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•8.11. (a) How does the node placement differ between Newton-Cotes quadrature and Clenshaw-Curtis quadrature? (b) Which would you expect to be more accurate for the same number of nodes? Why?

•8.12. (a) How does the node placement differ between Newton-Cotes quadrature and Gaussian quadrature?

•8.14. (a) Would you expect an *n*-point Newton-Cotes quadrature rule to work well for integrating Runge's function, $\int_{-1}^{1}(1+25x^2)^{-1}\,dx$ if *n* is very large? Why? (b) Same question for Chenshaw-Curtis quadrature rule.

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•8.15. (a) What is the degree of Simpson's rule for numerical quadrature? (b) What is the degree of an *n*-point Gaussian quadrature rule?

•8.16. Newton-Cotes and Gaussian quadrature rules are both based on polynomial interpolation. (a) What specific property characterizes a Newton-Cotes quadrature rule for a given number of nodes? (b) What specific property characterizes a Gaussian quadrature rule for a given number of nodes?

•8.17. (a) Explain how the midpoint rule, which is based on interpolation by a polynomial of degree zero, can nevertheless integrate polynomials of degree one exactly. (b) Is the midpoint rule a Gaussian quadrature rule? Explain your answer.

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•8.23. What is the relationship between Gaussian quadrature and orthogonal polynomials?

•8.24. (a) What does it meant for a sequence of quadrature rules to be progressive? (b) Why is this property important?

•8.25. (a) What is the advantage of using a Gauss-Kronrod pair of quadrature rules, such as G_7 and K_{15}, compared with using two Gaussian rules, such as G_7 and G_{15}, to obtain an approximate integral with error estimate? (b) How many evaluations of the integrand function are required to evaluate *both* of the rules G_7 and K_{15} in a given interval?

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•8.27. (a) What is a composite quadrature rule? (b) Why is a composite quadrature rule preferable to an ordinary quadrature rule for achieving high accuracy in numerically computing a definite integral on a given interval? (c) In using the composite trapezoid quadrature rule to approximate a definite integral on an interval [*a, b*], by what factor is the overall error reduced if the mesh size (i.e.subinterval length) *h* is halved?

•8.29. What is the most efficient way to use an adaptive quadrature routine for computing a definite integral whose integrand has a known discontinuity within the interval of integration?

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•8.30. What is a good way to integrate tabular data (i.e., an integrand whose value is known only at a discrete set of points)?

•8.32. How might one use a standard one-dimensional quadrature routine to compute the value of a double integral over a rectangular region?

•8.34. Relative to other methods for numerical quadrature, why is the Monte Carlo method more effective in higher dimensions than in low dimensions?

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•8.35. Explain why integral equations of the first kind with smooth kernels are always ill-conditioned.

•8.36. Explain how a quadrature rule can be used to solve an integral equation numerically. What type of computational problem results?

•8.38. List three approaches for obtaining a meaningful solution to an ill-conditioned linear system approximating an integral equation of the first kind.

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•8.41. (a) Suggest a good method for numerically approximating the derivative of a function whose value is given only at a discrete set of data points. (b) For this problem, what would be the effect of noisy data, and how would you cope with it in your numerical method?

•8.44. (a) Explain the basic idea of Richardson extrapolation. (b) Does it give a more accurate answer than the values on which it is based? (c) Does extrapolation to step size zero mean that the result is exact (i.e., the error is zero)?

•8.45. What is meant by Romberg integration?