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True or false:

•A linear least squares problem always has a solution,

•Fitting a straight line to a set of data points is a linear least squares problem, whereas fitting a quadratic polynomial to the data is a nonlinear least squares problem,

•At the solution to a linear least squares problem $Ax \approx b$, the residual vector $r = b - Ax$ is orthogonal to span(A),

•Methods for solving linear least square based on orthogonal factorization are more computationally expensive than the normal equations.

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•In a data-fitting problem in which *m* data points *(ti, yi)* are fit by a model function *f(t,x),* where t is the independent variable and x is an n-vector of parameters to be determined, what does it mean for the function *f* to be *linear* in the components of x?

•Give an example of a linear and nonlinear model function *f(t,x)*.

•In an overdetermined linear least squares problem with model function $f(t,x) = x_1\phi_1(t) + x_2\phi_2(t) + x_3\phi_3(t)$, what will be the rank of the resulting least squares matrix A if we take $\phi_1(t)=1$, $\phi_2(t)= t$, and $\phi_3(t)=1-t$?

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•What is the system of normal equations for the linear least squares problem $Ax \approx b$?

•Why are orthogonal transformations, such as Householder or Givens, often used to solve least squares problems?

•Why are such methods not often used to solve square linear systems?

•Do orthogonal transformations have any advantage over Gaussian elimination for solving square linear systems? If so, state one.

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•Which of the following properties does an $n \times n$ orthogonal matrix necessarily have?

(a)It is nonsingular.
(b)It preserves the Euclidean vector norm.
(c)Its transpose is its inverse.
(d)Its columns are orthonormal.
(e)It is symmetric.
(f)It is diagonal.
(g)Its Euclidean matrix norm is 1.
(h)Its Euclidean condition number 1.

•Show that multiplication by an orthogonal matrix preserves the Euclidean norm of a vector.

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•List one advantage and one disadvantage of Givens rotations for QR factorization compared with Householder transformations.

•When used to annihilate the second component of a 2-vector, does a Householder transformation always give the same result as a Givens rotation?

•Compared to the classical Gram-Schmidt procedure, which of the following are advantages of modified Gram-Schmidt orthogonalization?

(a)Requires less storage
(b) Requires less work
(c) Is more stable numerically.

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•In terms of the condition number of the matrix *A* compare the range of applicability of the normal equations method and the Householder QR method for solving the linear least squares problem $Ax \approx b$ [i.e., for what values of cond(*A*) can each method be expected to break down?].

•Let *A* be an $m \times n$ matrix:

(a) What is the maximum number of nonzero singular values that *A* can have?

(b) If rank$(A) = k$, how many nonzero singular values does *A* have?

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•Express the Euclidean condition number of a matrix in terms of its singular values.

•If A is a $2n \times n$ matrix, rank the following methods according to the amount of work required to solve the linear least squares problem $Ax \approx b$.

(a) QR factorization by Householder transformations

(b) Normal equations

(c) Singular value decomposition.

•List at least two applications for the singular value decomposition (SVD) of a matrix other then solving least squares problems.