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x.xx. Give four equivalent conditions for the non-singularity of an nxn matrix A.

x.xx. How many solutions can have a linear system $Ax = b$?

2.31. (a) State one defining property of a *singular* matrix A.

(b) Suppose that the linear system $Ax = b$ has two distinct solutions x and y. Use the property you gave in part *a* to prove that A must be singular.

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x.xx. Give the definition of the vector p-norm and particularly for p=1, 2 and Inf.

x.xx. Give three properties of any vector p-norm.

2.51. In 2-D, is it possible to have two vectors x and y that $||x||_1>||y||_1$ but $||x||_\infty<||y||_\infty$? Give an example.

x.xx. Given a norm of vector x, give the definition of the matrix norm for an nxn matrix A, and particularly for p=1 and ∞.

x.xx. Give five properties of any matrix norm.

2.22-2.24. True or false: (a) The norm of a singular matrix is zero; (b) if $||A|| = 0$, then $A = 0$; (c) $||A||_1=||A^T||_\infty$.

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x.xx. (a) How is the condition number of a matrix A defined for a given matrix norm?

2.25. (b) True or false: If A is any $n \times n$ nonsingular matrix, then $\text{cond}(A) = \text{cond}(A^{-1})$.

x.xx (c) Write the relation between the relative change in the solution, and the relative change in the problem data and the condition number.

x.xx. (d) How is the condition number used in estimating the accuracy (the number of accurate digits) of a computed solution to a linear system $Ax = b$?

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2.63. (a) In solving a linear system Ax = b, what is meant by the residual of an approximate solution x'?

x.xx. (b) How the relative change in the solution is bounded by the relative residual and the condition number of A?

(c) Does a small relative residual always imply that the solution is accurate? Why?

(d) Does a large relative residual always imply that the solution is inaccurate? Why?

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True or false:

2.1. If a matrix A is nonsingular, then the number of solutions to the linear system $Ax = b$ depends on the particular choice of right-hand-side vector b,

2.6. An underdetermined system of linear equations $Ax = b$ where A is an $m \times n$ matrix with $m < n$, always has a solution,

2.26. In solving a nonsingular system of linear equations, Gaussian elimination with partial pivoting usually yields a small residual even if the matrix is ill-conditioned.

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2.35. Specify an elementary elimination matrix that zeros the last two components of the vector: [3 2 -1 4]'.

2.36. Specify a 4 x 4 permutation matrix that interchanges the 2nd and 4th components of any 4-vector.

2.39. Consider the following matrix:

A = [4 -8 1;6 5 7;0 -10 -3], whose LU factorization we wish to compute using Gaussian elimination: What will the initial pivot element be if:

(a) No pivoting is used?

(b) Partial pivoting is used?

(c) Complete pivoting is used?

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2.45. If A and B are $n \times n$ matrices, with A nonsingular, and c is an n-vector, how would you efficiently compute the product $A^{-1}Bc$?

2.46. In a floating-point system having 10 decimal digits of precision, if Gaussian elimination with partial pivoting is used to solve a linear system whose matrix has a condition number of 10^3, and whose input data are accurate to full machine precision, about how many digits of accuracy would you expect in the solution?

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2.74. List three advantages of Cholesky factorization compared with LU factorization.

2.77. What is the Cholesky factorization of the following matrix: [4 2;2 2] ?

2.80. Suppose you have already solved the $n \times n$ linear system $Ax = b$ by LU factorization and back-substitution. What is the further cost (order of magnitude) of solving a new system:

(a) With the same matrix A but a different right-hand-side vector?

(b) With the matrix changed by adding a matrix of rank one?

(c) With the matrix A changed completely?