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**Homework Number:** 5
**Homework Title:** Exercise 5.7

## Problem description:

The gamma function has the following known values: $\Gamma(0.5) = \sqrt{\pi}$, $\Gamma(0.75) = 1.2254$, $\Gamma(1) = 1$. From these three values, determine the approximate value $x$ for which $\Gamma(x) = 1.5$, using one step of each of the following methods.

**(a)** Quadratic interpolation

**(b)** Inverse quadratic interpolation

**(c)** Linear fractional interpolation

## Problem solution:

**(a) Quadratic interpolation:** For quadratic interpolation we first need the interpolation polynomial for three supporting points. I used the metode of Lagrange with equidistant supporting points: $x_1 = 0.5, x_2 = 0.75, x_3 = 1$ and theire values of the gamma function: $y_1 = \sqrt{\pi}, y_2 = 1.2254, y_3 = 1$.

$$p(x) = y_1 \frac{x - x_2}{x_1 - x_2} \frac{x - x_3}{x_1 - x_2} + y_2 \frac{x - x_1}{x_2 - x_3} \frac{x - x_3}{x_2 - x_3} + y_3 \frac{x - x_1}{x_3 - x_1} \frac{x - x_2}{x_3 - x_2}$$

$$\Rightarrow p(x) = 55.97x^2 - 89.04x + 34.07$$

To get the aproximate value $x$ where $p(x) = 1.5$ we have to find the roots of the polynomial $p(x) - 1.5 = 0$:

$$p(x) - 1.5 = 55.97x^2 - 89.04x + 32.57 = 0$$

We get the values $x_1 = 0.5702$ and $x_2 = 1.021$. From the values around we can conclude that $x_1$ is our requested value.

**(b) Inverse quadratic interpolation:** For the following two methods we need the starting values $a_0 = 0.5, b_0 = 0.75, c_0 = 1$ and the function $f(x) = \Gamma(x) - 1.5$ with $f(a_0) = 0.2725, f(b_0) = -0.2746$ and $f(c_0) = -0.5$.

$$u = \frac{f(b_0)}{f(c_0)} = 0.5492 \qquad v = \frac{f(b_0)}{f(a_0)} = -1.008 \qquad w = \frac{(a_0)}{(c_0)} = -0.545$$

$$p = v(w(u-w)(c-b) - (1-u)(b-a)) = 0.2638$$

$$q = (w-1)(u-1)(v-1) = -1.398$$

$$\Rightarrow b_1 = b_0 + \frac{p}{q} = 0.5614$$

**(c) Linear fractional interpolation:**

$$c_1 = c + \frac{(a-c)(b-c)(f(a)-f(b))f(c)}{(a-c)(f(c)-f(b))f(a) - (b-c)(f(c)-f(a))f(b)} = 0.5917$$

## Results:

| **Method** | Iterations | $\tilde{x}$ | $\Gamma(x)$ | $\lvert\tilde{x} - x\rvert$ |
|---|---|---|---|---|
| Quadratic interpolation | - | 0.5702 | 1.562 | 0.025 |
| Inverse quadratic interpolation | 1 | 0.5614 | 1.585 | 0.034 |
| Linear fractional interpolation | 1 | 0.5917 | 1.509 | 0.0036 |