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Homework number: 3
Homework Title: Exercise 3.25

Problem description:
Verify that the dominant term in the operation count (number of multiplications or number of additions) for QR factroization of an m x n matrix using Householder transformation is $nm^2 - n^3/3$.

Problem solution:
The Householder Transformation is an algorithm which performs QR factorization for a given m x n matrix A. A possible implementation of the algorithm could look like this:

```
for k=1:n
  no=norm(ak)              % m add/sub
  v=a-no*ek                % m add/sub
  p=v'*v                   % m add/sub
  for j=k:n
    g=(2*v'*aj)/p          % m-k add/sub
    for i=k:m
      A[i,j]=A[i,j]-g*v[i]  % 1 add/sub
    end
  end
end
```

If we count the approximate number of additions/subtraction of the single steps, we finally can calculate the approximate number of floting point operations (flops in short) in the following way:

$$flops \cong \sum_{k=1}^{n} \left(3m + \underbrace{\sum_{j=k}^{n} \left(m - k + \underbrace{\sum_{i=k}^{m} 1}_{s_1} \right)}_{s_2} \right)$$

where

$$s_1 = \sum_{i=k}^{m} 1 = m - k + 1$$

and

$$s_2 = \sum_{j=k}^{n} \left(m - k + \sum_{i=k}^{m} 1 \right) =$$

$$\begin{aligned}
&= \sum_{j=k}^{n} (m-k+m-k+1) = \\
&= (2m-2k+1)(n-k+1) = \\
&= 2mn - 2km + 2m - 2kn + 2k^2 - 2k + n - k + 1 = \\
&= 2k^2 - k(2m+2n+3) + 2mn + 2m + n + 1
\end{aligned}$$

For the total sum we get:

$$\begin{aligned}
flops &\cong \sum_{k=1}^{n} \left(3m + \sum_{j=k}^{n} \left(m - k + \sum_{i=k}^{m} 1 \right) \right) = \\
&= \sum_{k=1}^{n} \left(3m + 2k^2 - k(2m+2n+3) + 2mn + 2m + n + 1 \right) = \\
&= 2\sum_{k=1}^{n} k^2 - (2m+2n+3) \sum_{k=1}^{n} k + (2mn + 5m + n + 1) \sum_{k=1}^{n} 1 = \\
&= 2\frac{n(n+1)(2n+1)}{6} - (2m+2n+3)\frac{n(n+1)}{2} + n(2mn+5m+n+1) = \\
&= \frac{2n^3}{3} + n^2 + \frac{n}{3} - mn^2 - mn - n^3 - n^2 - \frac{3n^2}{2} - \frac{3n}{2} + 2mn^2 + 5mn + n^2 + n = \\
&= mn^2 - \frac{n^3}{3} - \frac{n^2}{2} + 4mn - \frac{n}{6}
\end{aligned}$$

For big m and n we can simplify the term and get approximately $mn^2 - \frac{n^3}{3}$ floating point operations for the Householder transformation.