First name: Robert
Last Name: Resch
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Homework number: 2
Homework Title: Exercise 2.13

Problem description:
How would you solve a partitioned linear system of the form

$$\begin{bmatrix} L_1 & O \\ B & L_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b \\ c \end{bmatrix},$$

where L_1 and L_2 are nonsingular lower triangular matrices, and the solution and right-hand-side vectors are partitioned accordingly? Show the specific steps you would perform in terms of the given submatrices and vectors.

Problem solution:
First of all I would solve the equation $\begin{bmatrix} L_1 & O \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b \end{bmatrix}$. Since the right part of the equation matrix is the zero matrix O it's sufficient to solve the linear system $L_1 x = b$.
But as we know, L_1 is a nonsingular lower triangular matrix which we can solve using Forward-Substitution (see the belonging mathimatical expression below).

$$x_1 = b_1/l_{11},\ x_i = \left(b_i - \sum_{j=1}^{i-1} l_{ij} x_j \right) / l_{ii},\ i = 2, \ldots, n$$

Afterwards I would solve the system $\begin{bmatrix} B & L_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c \end{bmatrix}$ which we could write as $Bx + L_2 y = c$. But as we already know the vector x from the previous step, we can easily compute the vector Bx. So we only have to solve the linear system $L_2 y = c - Bx$ where L_2 is a nonsingular lower triangular matrix. Thus, we can solve this linear equation by Forward-Substitution and get the vector y. Finally, we have found the solution for the linear system

$$\begin{bmatrix} L_1 & O \\ B & L_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b \\ c \end{bmatrix},$$

Results:
As we can see in the example aboth, we can bring a nonsingular lower triangular matrix in the form

$$\begin{bmatrix} L_1 & O \\ B & L_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b \\ c \end{bmatrix},$$

and solve it with the described algorithm.