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Homework Number: 2
Homework Title: Exercise 2.12

Problem description:

Verify that the dominant term in the operation count (number of multiplications or number of additions) for solving a lower triangular system of order n by forward substitution is $\frac{n^2}{2}$.

Problem solution:

We have a triangular system $L \cdot x = b$

$$\begin{pmatrix} l_{11} & 0 & 0 & 0 & 0 \\ . & . & 0 & 0 & 0 \\ l_{k1} & . & l_{kk} & 0 & 0 \\ . & . & . & . & 0 \\ l_{n1} & . & l_{nk} & . & l_{nn} \end{pmatrix} \cdot x = b$$

To solve this system by forward substitution we can use the following algorithm

```
for j = 0 to n
    x_j = b_j / l_jj
    for i = j + 1 to n
        b_i = b_i - l_ij x_j
    end
end
```

To get the number of multiplications(divisions) for the algorithm to finish can be descriped by following equation

$$N = n + \sum_{k=1}^{n} (n - k) \tag{1}$$

The first n comes from the division $x_j = b_j / l_{jj}$ in the outer loop. The sum $(n - k)$ originates from the inner loop allowing for the outer for. By simplifying this formula we get

$$N \quad = \quad n + \sum_{k=0}^{n-1} k$$

$$\begin{aligned}
N &= n + \frac{n\,(n-1)}{2} \\
N &= n + \frac{n^2 - n}{2} \\
N &= \frac{n^2}{2} + \frac{n}{2}
\end{aligned}$$

To get the number of additions(subtractions) we can use formula 1 without the first term n because there is no addition in the outer loop but also one subtraction in the inner one. After simplifying we get

$$N = \frac{n^2}{2} - \frac{n}{2}$$

Results:

1. Number of multiplications(divisions)

$$N = \frac{n^2}{2} + \frac{n}{2}$$

2. Number of additions(subtractions)

$$N = \frac{n^2}{2} - \frac{n}{2}$$

For both - multiplication and addition - the dominant term of there operation count is $\frac{n^2}{2}$.