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Date: 28.10.04
Homework number: 1
Homework Title: Exercise 1.11

Problem description:
If x≈y, then we would expect some cancellation in computing log(x) - log(y). On the other hand, log(x) - log(y) = log(x/y), and the latter involves no cancellation. Does this mean that computing log(x/y) is likely to give a better result? (Hint: For what value is the log function sensitive?)

Problem solution:
As $ln(x)$ is a continuous function, $ln(x) \approx ln(y)$ for $x \approx y$. So in this case, the result of the operation $ln(x) - ln(y)$ may suffer from some cancellation, due to the subtraction of two approximatly equal numbers.
As we know, $ln(x) - ln(y) = ln(\frac{x}{y})$. The question is, if computing $ln(\frac{x}{y})$ will give a better result than $ln(x) - ln(y)$. To answer this question we must have a look at the $ln(x)$ function in more detail.
First of all let's have a look at the sensitivity of the function or more quantitatively at the condition number. As we know we can approximate the condition number in the following way:

$$ConditionNumber \approx \left|\frac{xln'(x)}{ln(x)}\right| = \left|\frac{x\frac{1}{x}}{ln(x)}\right| = \left|\frac{1}{ln(x)}\right|.$$

Thus we can observe, that low $ln(x)$ results in a high condition number and so maybe an erronous result. The condition number will be extremly high for $ln(x) \to 0$ that means $x \to 1$. But what does that mean in our case, where $x \approx y$?
As $x \approx y \Rightarrow \frac{x}{y} \approx 1$, so $ln(\frac{x}{y})$ will be extremly ill-conditioned. So we have solved the cancellation problem at the cost of a very ill-conditioned algorithm.

Results:
$x \approx y \Rightarrow ln(x) \approx ln(y)$ and so $ln(x) - ln(y)$ suffers from some cancellation. On the other hand $x \approx y \Rightarrow \frac{x}{y} \approx 1 \Rightarrow ln(\frac{x}{y}) \approx 0$ resulting in an extremly ill-conditioned algorithm. Thus, $ln(\frac{x}{y})$ is not likely to give a seriously better result than $ln(x) - ln(y)$ because the cancellation problem was displaced by the problem of an ill-conditioned algorithm.