

REVIEW - CHAPTER 8

- 8.1. True or false: Evaluating a definite integral is always a well-conditioned problem.
- 8.2. True or false: Because it is based on polynomial interpolation of degree one higher, the trapezoid rule is generally more accurate than the midpoint rule.
- 8.4. True or false: An n -point Newton-Cotes quadrature rule is always of degree $n - 1$.
- 8.5. True or false: Gaussian quadrature rules of different orders never have any points in common.

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- 8.9. Name two different methods for computing the weights corresponding to a given set of nodes of a quadrature rule.
- 8.10. How can you estimate the error in a quadrature rule without computing the derivatives of the integrand function that would be required by a Taylor series expansion?

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- 8.11. (a) How does the node placement differ between Newton-Cotes quadrature and Clenshaw-Curtis quadrature? (b) Which would you expect to be more accurate for the same number of nodes? Why?
- 8.12. (a) How does the node placement differ between Newton-Cotes quadrature and Gaussian quadrature?
- 8.14. (a) Would you expect an n -point Newton-Cotes quadrature rule to work well for integrating Runge's function, $\int_{-1}^1 (1+25x^2)^{-1} dx$ if n is very large? Why? (b) Same question for Clenshaw-Curtis quadrature rule.

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- 8.15. (a) What is the degree of Simpson's rule for numerical quadrature? (b) What is the degree of an n -point Gaussian quadrature rule?
- 8.16. Newton-Cotes and Gaussian quadrature rules are both based on polynomial interpolation. (a) What specific property characterizes a Newton-Cotes quadrature rule for a given number of nodes? (b) What specific property characterizes a Gaussian quadrature rule for a given number of nodes?
- 8.17. (a) Explain how the midpoint rule, which is based on interpolation by a polynomial of degree zero, can nevertheless integrate polynomials of degree one exactly. (b) Is the midpoint rule a Gaussian quadrature rule? Explain your answer.

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- 8.23. What is the relationship between Gaussian quadrature and orthogonal polynomials?
- 8.24. (a) What does it mean for a sequence of quadrature rules to be progressive? (b) Why is this property important?
- 8.25. (a) What is the advantage of using a Gauss-Kronrod pair of quadrature rules, such as G_7 and K_{15} , compared with using two Gaussian rules, such as G_7 and G_{15} , to obtain an approximate integral with error estimate? (b) How many evaluations of the integrand function are required to evaluate *both* of the rules G_7 and K_{15} in a given interval?

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- 8.27. (a) What is a composite quadrature rule? (b) Why is a composite quadrature rule preferable to an ordinary quadrature rule for achieving high accuracy in numerically computing a definite integral on a given interval? (c) In using the composite trapezoid quadrature rule to approximate a definite integral on an interval $[a, b]$, by what factor is the overall error reduced if the mesh size (i.e. subinterval length) h is halved?
- 8.29. What is the most efficient way to use an adaptive quadrature routine for computing a definite integral whose integrand has a known discontinuity within the interval of integration?

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- 8.30. What is a good way to integrate tabular data (i.e., an integrand whose value is known only at a discrete set of points)?
- 8.32. How might one use a standard one-dimensional quadrature routine to compute the value of a double integral over a rectangular region?
- 8.34. Relative to other methods for numerical quadrature, why is the Monte Carlo method more effective in higher dimensions than in low dimensions?

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- 8.35. Explain why integral equations of the first kind with smooth kernels are always ill-conditioned.
- 8.36. Explain how a quadrature rule can be used to solve an integral equation numerically. What type of computational problem results?
- 8.38. List three approaches for obtaining a meaningful solution to an ill-conditioned linear system approximating an integral equation of the first kind.

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- 8.41. (a) Suggest a good method for numerically approximating the derivative of a function whose value is given only at a discrete set of data points. (b) For this problem, what would be the effect of noisy data, and how would you cope with it in your numerical method?
- 8.44. (a) Explain the basic idea of Richardson extrapolation. (b) Does it give a more accurate answer than the values on which it is based? (c) Does extrapolation to step size zero mean that the result is exact (i.e., the error is zero)?
- 8.45. What is meant by Romberg integration?