REVIEW - CHAPTER 7

•7.1. True or false: There are arbitrarily many different mathematical functions that interpolate a given set of data points.

•7.2. True or false: If an interpolating function accurately reproduces the given data values, then this fact implies that the coefficients in the linear combination of basis functions are well-determined.

•7.3. True or false: If the polynomial interpolating a given set of data points is unique, then so is the representation of that polynomial.

•7.4. True or false: When interpolating a continuous function by a polynomial at equally spaced points on a given interval, the polynomial interpolant always converges to the function as the number of interpolation points increases.

REVIEW - CHAPTER 7

•7.5. What is the basic distinction between interpolation and approximation of a function?

•7.6. State at least two different applications for interpolation.

•7.8. Is it ever possible for two distinct polynomials to interpolate the same *n* data points? If so, under what conditions, and if not, why?

•7.9. State at least two important criteria for choosing a particular set of basis functions for use in interpolation.

REVIEW - CHAPTER 7

•7.10. Determining the parameters of an interpolant can be interpreted as solving a linear system $Ax = y$, where the matrix A depends on the basis functions used and the vector y contains the function values to be fit. Describe in words the pattern of nonzero entries in the matrix A for polynomial interpolation using each of the following bases: (a) Monomial basis (b) Lagrange basis (c) Newton basis.

•7.11. (a) Is interpolation an appropriate procedure for fitting a function to noisy data? (b) If so, why, and if not, what is a good alternative?

•7.13. (a) What is a Vandermonde matrix? (b) In what context does such a matrix arise? (c) Why is such a matrix often ill-conditioned when its order is relatively large?

REVIEW - CHAPTER 7

•7.16. List one advantage and one disadvantage of Lagrange interpolation compared with using the monomial basis for polynomial interpolation.

•7.18. Why is interpolation by a polynomial of high degree often unsatisfactory?

•7.19. (a) In interpolating a continuous function by a polynomial, what key features determine the error in approximating the function by the resulting interpolant? (b) Under what circumstances can the error be large even though the number of interpolation points is large?

REVIEW - CHAPTER 7

•7.20. How should the interpolation points be placed in an interval in order to guarantee convergence of the polynomial interpolant to sufficiently smooth functions on the interval as the number of points increases?

•7.21. What does it mean for two polynomials p and q to be *orthogonal* to each other on an interval $[a, b]$?

•7.22. (a) What is meant by a *Taylor* polynomial? (b) In what sense does it interpolate a given function?

•7.23. In fitting a large number of data points, what is the main advantage of piecewise polynomial interpolation over interpolation by a single polynomial?

REVIEW - CHAPTER 7

•7.24. (a) How do Hermite and spline interpolations differ from ordinary interpolation?

•7.25. In choosing between Hermite cubic and cubic spline interpolation, which should one choose (a) If maximum smoothness of the interpolant is desired? (b) If the data are monotonic and this property is to be preserved?

•7.26. (a) How many times is a Hermite cubic interpolant continuously differentiable? (b) How many times is a cubic spline interpolant continuously differentiable?

REVIEW - CHAPTER 7

•7.29. Which of the following interpolants to n data points are unique? (a) Polynomial of degree at most n-1 (b) Hermite cubic (c) Cubic spline

•7.30. For which of the following types of interpolation is it possible, in general, to preserve monotonicity in a set of n data points (i.e., the interpolant is increasing or decreasing if the data points are increasing or decreasing)? (a) Polynomial of degree at most n - 1 (b) Hermite cubic (c) Cubic spline.

•7.31. Why is it advantageous if the basis functions used for interpolation are localized (i.e., each basis function involves only a few data points)?