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• 5.1. True or false: A small residual $||f(x)||$ guarantees an accurate solution of a system of nonlinear equations $f(x)=0$.

•5.2. True or false: Newton's method is an example of a fixed-point iteration scheme.

•5.3. True or false: If an iterative method for solving a nonlinear equation gains more than one bit of accuracy per iteration, then it is said to have a superlinear convergence rate.

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• 5.5. Suppose you are using an iterative method to solve a nonlinear equation $f(x) = 0$ for a root that is ill-conditioned, and you need to choose a convergence test. Would it be better to terminate the iteration when you find an iterate x_k for which $||f(x_k)||$ is small, or when $|| x_k - x_{k-1} ||$ is small? Why?

•5.6. (a) What is meant by a bracket for a nonlinear function in one dimension?(b) What does this concept have to do with zero finding?

•5.7. For root finding problems, why must we use an absolute rather than a relative condition number in assessing sensitivity?

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• 5.9. If the errors at successive iterations of an iterative method are as follows, how would you characterize the convergence rate?

–(a)10^{-2}, 10^{-4}, 10^{-8}, 10^{-16}, . . .

–(b))10^{-2}, 10^{-4}, 10^{-6}, 10^{-8}, . . .

•5.12. Suppose you are using the bisection method to find a zero of a nonlinear function, starting with an initial bracketing interval $[a, b]$. Give a general expression for the number of iterations that will be required to achieve an error tolerance of tol for the length of the final bracketing interval.

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• 5.13. What is meant by a *quadratic* convergence rate for an iterative method?

•5.14. If an iterative method squares the error every *two* iterations, what is its convergence rate r ?

•5.15. (a) What does it mean for a root of an equation to be a *multiple* root?

(b) What is the effect of a multiple root on the convergence rate of the bisection method?

(c) What is the effect of a multiple root on the convergence rate of Newton's method?

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• 5.17. What is the convergence rate for Newton's method for finding the root x = 2 of each of the following equations? (a)$f(x) = (x - 1)(x - 2)^2 = 0$; (b)$f(x) = (x - 1)^2(x - 2) = 0$

5.20. Let g: $\mathbf{R} \rightarrow \mathbf{R}$ be a smooth function having a fixed point x^*. (a)What condition determines whether the iteration scheme $x_{k+1} = g(x_k)$ is locally convergent to x^*? (b)What is the convergence rate? (c)What additional condition implies that the convergence rate is quadratic? (d)Is Newton's method for finding a zero of a smooth function $f : \mathbf{R} \rightarrow \mathbf{R}$ an example of such a fixed-point iteration scheme? If so, what is the function g in this case? If not, then explain why not.

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• 5.26. For solving a one-dimensional nonlinear equation, how many function or derivative evaluations are required per iteration of each of the following methods? (a) Newton's method (b) Secant method

•5.27. Rank the following methods 1 through 3, from slowest convergence rate to fastest convergence rate, for finding a simple root of a nonlinear equation in one dimension:(a)Bisection method (b)Newton's method (c)Secant method

•5.28. In solving a nonlinear equation in one dimension, how many bits of accuracy are gained per iteration of (a) Bisection method? (b) Newton's method?

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• 5.30. What is meant by inverse interpolation? Why is it useful for root finding problems in one dimension?

•5.31. Suppose that you are using fixed-point iteration based on the fixed-point problem $x=g(x)$ to find a solution x^* to a nonlinear equation $f(x) = 0$. Which would be more favorable for the convergence rate: a horizontal tangent of g at x^* or a horizontal tangent of f at x^*? Why?

•5.33. For what type of function is linear fractional interpolation a particularly good choice of zero finder?

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5.35. State at least one method for finding all the zeros of a polynomial, and discuss its advantages and disadvantages.

5.37. For solving an n-dimensional nonlinear equation, how many scalar function evaluations are required per iteration of Newton's method?

5.39. Give two reasons why secant updating methods for solving systems of nonlinear equations are often more efficient than Newton's method despite converging more slowly.