REVIEW - CHAPTER 4

•4.2. True or false: All the eigenvalues of a real matrix are necessarily real.

•4.3. True or false: An eigenvector corresponding to a given eigenvalue of a matrix is unique.

•4.4. True or false: Every $n \times n$ matrix A has n linearly independent eigenvectors.

•4.6. True or false: A square matrix A is singular if, and only if, 0 is one of its eigenvalues.

•4.9. True or false: The eigenvalues of a complex Hermitian matrix must be real.

REVIEW - CHAPTER 4

• 4.11. True or false: If two matrices are similar, then they have the same eigenvectors.

•4.12. True or false: Given any arbitrary square matrix, there is some diagonal matrix that is similar to it.

•4.13. True or false: Given any arbitrary square matrix, there is some triangular matrix that is unitarily similar to it.

REVIEW - CHAPTER 4

• 4.15. True or false: The eigenvalues of a real symmetric or complex Hermitian matrix are always well-conditioned.

•4.16. True or false: A matrix that is both symmetric and Hessenberg must be tridiagonal.

•4.17. True or false: If an $n \times n$ matrix A has distinct eigenvalues, then QR iteration applied to A necessarily converges to a diagonal matrix.

•4.18. True or false: For a square matrix, the eigenvalues and the singular values are the same.

REVIEW - CHAPTER 4

• 4.19. Explain the distinction between a right eigenvector and a left eigenvector.

•4.20. What is meant by the spectral radius of a matrix?

•4.22. What is meant by the characteristic polynomial of a matrix? What does it have to do with eigenvalues?

•4.24. What is meant by an invariant subspace for a given matrix A?

•4.27. Which of the following classes of matrices necessarily have all real eigenvalues? (a) Real symmetric (b) Real triangular (c) Arbitrary real (d) Complex symmetric (e) Complex Hermitian (f) Complex triangular with real diagonal (g) Arbitrary complex

REVIEW - CHAPTER 4

• 4.30. The eigenvalues of a matrix are the roots of its characteristic polynomial. Does this fact provide a generally effective numerical method for computing the eigenvalues? Why?

•4.32. A general matrix can be reduced to triangular form by a single QR factorization, and the eigenvalues of a triangular matrix are its diagonal entries. Does this procedure suffice to compute the eigenvalues of the original matrix? Why?

•4.33. Gauss-Jordan elimination reduces a matrix to diagonal form. Does this make the eigenvalues of the matrix obvious? Why?

REVIEW - CHAPTER 4

•4.37. Applied to a given matrix A, QR iteration for computing eigenvalues converges to either diagonal or triangular form. What property of A determines which of these two forms is obtained?

•4.38. As a preliminary step before computing its eigenvalues, a matrix A is often first reduced to Hessenberg form by a unitary similarity transformation. Why stop there? If such a preliminary reduction to Hessenberg form is good, wouldn't triangular form be even better? What is wrong with this argument?

REVIEW - CHAPTER 4

•4.39. If you had a routine for computing all the eigenvalues of a non-symmetric matrix, how could you use it to compute the roots of any polynomial?

•4.42. (a) If a matrix *A* has a simple dominant eigenvalue λ_1, what quantity determines the convergence rate of the power method for computing λ_1?

(b) How can the convergence rate of power iteration be improved?

REVIEW - CHAPTER 4

•4.43. Given an approximate eigenvector *x* for a matrix *A*, what is the best estimate (in the least squares sense) for the corresponding eigenvalue?

•4.47. What is the main reason that shifts are used in iterative methods for computing eigenvalues, such as the power, inverse iteration, and QR iteration methods?

•4.48. Given a general square matrix *A*, what method would you use to compute the following?

•(a) The smallest eigenvalue of *A* (b) The largest eigenvalue of *A* (c) The eigenvalue of *A* closest to some specified scalar *b*(d) All of the eigenvalues of *A*

REVIEW - CHAPTER 4

•4.56. (a) List two reasons why converting a generalised eigenvalue problem $Ax = lBx$ to the standard eigenvalue problem $(B^{-1} A)x = lx$ might not be a good idea. (b) What is a better approach?

•4.57. How are the singular values of an *m ´ n* real matrix *A* related to the eigenvalues of the *n ´ n* matrix $A^T A$?