

REVIEW - CHAPTER 3

True or false:

- A linear least squares problem always has a solution,
- Fitting a straight line to a set of data points is a linear least squares problem, whereas fitting a quadratic polynomial to the data is a nonlinear least squares problem,
- At the solution to a linear least squares problem $Ax \approx b$, the residual vector $r = b - Ax$ is orthogonal to $\text{span}(A)$,
- Methods for solving linear least square based on orthogonal factorization are more computationally expensive than the normal equations.

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- In a data-fitting problem in which m data points (t_i, y_i) are fit by a model function $f(t, x)$, where t is the independent variable and x is an n -vector of parameters to be determined, what does it mean for the function f to be *linear* in the components of x ?
- Give an example of a linear and nonlinear model function $f(t, x)$.
- In an overdetermined linear least squares problem with model function $f(t, x) = x_1 f_1(t) + x_2 f_2(t) + x_3 f_3(t)$, what will be the rank of the resulting least squares matrix A if we take $f_1(t) = 1$, $f_2(t) = t$, and $f_3(t) = 1 - t$?

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- What is the system of normal equations for the linear least squares problem $Ax \approx b$?
- Why are orthogonal transformations, such as Householder or Givens, often used to solve least squares problems?
- Why are such methods not often used to solve square linear systems?
- Do orthogonal transformations have any advantage over Gaussian elimination for solving square linear systems? If so, state one.

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- Which of the following properties does an $n \times n$ orthogonal matrix necessarily have?
 - (a) It is nonsingular.
 - (b) It preserves the Euclidean vector norm.
 - (c) Its transpose is its inverse.
 - (d) Its columns are orthonormal.
 - (e) It is symmetric.
 - (f) It is diagonal.
 - (g) Its Euclidean matrix norm is 1.
 - (h) Its Euclidean condition number is 1.
- Show that multiplication by an orthogonal matrix preserves the Euclidean norm of a vector.

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- List one advantage and one disadvantage of Givens rotations for QR factorization compared with Householder transformations.
- When used to annihilate the second component of a 2-vector, does a Householder transformation always give the same result as a Givens rotation?
- Compared to the classical Gram-Schmidt procedure, which of the following are advantages of modified Gram-Schmidt orthogonalization?
 - (a) Requires less storage
 - (b) Requires less work
 - (c) It is more stable numerically.

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- In terms of the condition number of the matrix A compare the range of applicability of the normal equations method and the Householder QR method for solving the linear least squares problem $Ax \approx b$ [i.e., for what values of $\text{cond}(A)$ can each method be expected to break down?].
- Let A be an $m \times n$ matrix:
 - (a) What is the maximum number of nonzero singular values that A can have?
 - (b) If $\text{rank}(A) = k$, how many nonzero singular values does A have?

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- Express the Euclidean condition number of a matrix in terms of its singular values.
- If A is a $2n \times n$ matrix, rank the following methods according to the amount of work required to solve the linear least squares problem $Ax \approx b$.
 - (a) QR factorization by Householder transformations
 - (b) Normal equations
 - (c) Singular value decomposition.
- List at least two applications for the singular value decomposition (SVD) of a matrix other than solving least squares problems.