Homework 7

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Exercise 8.6: Suppose that Lagrange interpolation at a given set of nodes $x_1, \ldots, x_n$ is used to derive a quadrature rule. Prove that the corresponding weights are given by the integrals of the Lagrange basis functions, $w_i = \int_a^b \ell_i(x)dx$, $i = 1, \ldots, n$.

Solution: When using Lagrange-Interpolation for deriving a quadrature rule an exact integral $\int_a^b f(x)dx$ is approximated by the integral over its interpolating polynomial in Lagrange form

$$\int_a^b p_n(x)dx$$

where $p_n(x) = \sum_{j=1}^{n+1} f(x_i)\ell_i(x)$. So the quadrature rule looks like

$$Q_n = \int_a^b p_n(x)dx = \int_a^b \sum_{j=1}^{n+1} f(x_i)\ell_i(x)dx = \sum_{j=1}^{n+1} f(x_i) \int_a^b \ell_i(x)dx = \sum_{j=1}^{n+1} f(x_i)w_i$$