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Problem Description

Given a sufficiently smooth function $f : \mathbb{R} \to \mathbb{R}$, use Taylor series to derive a second-order accurate, one-sided difference approximation to $f'(x)$ in terms of the values of $f(x)$, $f(x+h)$, and $f(x+2h)$.

Problem Solution

$$\begin{aligned}
f(x+h) &= f(x) + f'(x)h + \frac{f''(x)}{2}h^2 + \frac{f'''(x)}{6}h^3 + \dots \\
f(x+2h) &= f(x) + f'(x)2h + f''(x)2h^2 + \frac{f'''(x)}{6}8h^3 + \dots \\
4f(x+h) - f(x+2h) &= 3f(x) + 2f'(x)h - 4\frac{f'''(x)}{6}h^3 - \dots \\
f'(x) &= \frac{4f(x+h) - f(x+2h) - 3f(x)}{2h} - 2\frac{f'''(x)}{6}h^2 - \dots \\
f'(x) &\approx \frac{4f(x+h) - f(x+2h) - 3f(x)}{2h}
\end{aligned}$$

The approximation is second-order accurate since the dominant term in the remainder of the series is $\mathcal{O}(h^2)$.

Results

$$f'(x) \approx \frac{4f(x+h) - f(x+2h) - 3f(x)}{2h}$$