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**Homework Title:** Exercise 8.14

### Problem Description:

Suppose that the first-order accurate, forward difference approximation to the derivative of a function at a given point produces the value -0.8333 for  $h=0.2$  and the value -0.9091 for  $h=0.1$ . Use Richardson extrapolation to obtain a better approximate value for the derivative.

### Problem Solution:

The finite forward difference approximation to the derivative of a function  $f(h)$  is denoted as

$$F(h) = a_0 + a_1h + O(h^2) .$$

For  $h_1 = 0.2$  and  $h_2 = h_1/2 = 0.1$  the values  $F(h_1)$  and  $F(h_1/2)$  are already computed.

Inserting these values into the above equation, we get

$$F(h_1) = a_0 + a_1h_1 + O(h_1^2) , \text{ and}$$

$$F(h_1/2) = a_0 + a_1h_1/2 + O(h_1^2) .$$

Solving these two equations in the unknowns  $a_0$  and  $a_1$  gives

$$a_0 = F(h_1) + \frac{F(h_1) - F(h_1/2)}{1/2 - 1} + O(h^2) ,$$

where  $a_0$  is the value we are searching for.

$$\begin{aligned} a_0 &= F(h_1) + \frac{F(h_1) - F(h_1/2)}{1/2 - 1} \\ &= 2F(h_1/2) - F(h_1) \\ &= 2 \cdot (-0.9091) + 0.8333 \\ &= -0.9849 \end{aligned}$$

### Results:

$$a_0 = -0.9849$$