

First name: Franz-Josef
Last Name: Auernigg
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Homework number: 7
Homework Title: Exercise 8.1

Problem description:

- (a) Compute the approximate value of the integral $\int_0^1 x^3 dx$, first by the midpoint rule and then by the trapezoid rule.
- (b) Use the difference between these two results to estimate the error in each of them.
- (c) Combine the two results to obtain the Simpson's rule approximation to the integral.
- (d) Would you expect the latter to be exact for this problem? Why?

Problem solution:

The function f used for midpoint and trapezoid rule is $f(x) = x^3$

- (a) The Midpoint rule for the interval $[a,b]$ is defined as

$$M(f) = (b - a) * f\left(\frac{a + b}{2}\right) \quad (1)$$

For the interval $[1,0]$ we get: $M(f) = (1 - 0) * \left(\frac{1+0}{2}\right)^3 = \frac{1}{8}$

The trapezoid rule for the interval $[a,b]$ is defined as

$$T(f) = \frac{b - a}{2} * (f(a) + f(b)) \quad (2)$$

For the interval $[1,0]$ we get: $T(f) = \frac{1-0}{2} * (0 + 1) = \frac{1}{2}$

- (b) For error estimation I used the equation

$$E(f) = \frac{T(f) - M(f)}{3} \quad (3)$$

and

$$E_T(f) = -2 * E \quad (4)$$

$$E(f) = \frac{\frac{1}{2} - \frac{1}{8}}{3} = \frac{1}{8}$$

$$E_T(f) = -2 * \frac{1}{8} = -\frac{1}{4}$$

(c) Simpson's rule:

$$S(f) = \frac{b-a}{6} * (f(a) + 4 * f(\frac{a+b}{2}) + f(b)) \tag{5}$$

or

$$S(f) = \frac{2}{3} * M(f) + \frac{1}{3} * T(f) \tag{6}$$

I used the latter and got $S(f) = \frac{2}{3} * M(f) + \frac{1}{3} * T(f) = \frac{1}{4}$

(d) Yes. Simpson rule is of degree 3 and that's why it is expected to be exact.

Results:

- a) $M(f) = \frac{1}{8}$ and $T(f) = \frac{1}{2}$
- b) $E(f) = \frac{1}{8}$ and $E_T(f) = -\frac{1}{4}$
- c) Simpson's rule: $S(f) = \frac{2}{3} * M(f) + \frac{1}{3} * T(f) = \frac{1}{4}$
- d) Yes, because Simpson's rule is of degree 3.