Homework 6

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Exercise 7.8: Compare the cost of forming a Vandermonde matrix inductively, as in Section 7.3.1, with the cost using explicit exponentiation.

Solution: The cost for forming a single row of an $n \times n$-Vandermonde matrix $\mathbf{A}$ inductively is $n-2$ multiplications:

For t_i^2 1 multiplication is needed. Since $a_{i,j+1} = t_i a_{i,j}$, for each additional column another multiplication comes on. That means for 3 columns we need 1 multiplication (the exponent in this column is 2, see above), for 4 columns 2, for 5 columns 3, and so on. In general $n-2$ multiplications are needed for a row of a matrix with n columns. This has to be done for every row of the matrix, so the cost for forming the whole matrix inductively is

$$n(n-2)$$

.

When explicit exponentiation is used, the cost for forming a single row increases to $\sum_{j=1}^{n-2} j$ because now for the entry t^{j-1} in column j, one has to perform $j-2$ multiplications and all of the columns are treated separately. Again, this has to be done for every row, and so the cost for the whole matrix is

$$n \sum_{j=1}^{n-2} j.$$

When compared, latter method needs $n \sum_{j=1}^{n-3} j$ mulitplications more than the former:

$$n \sum_{j=1}^{n-2} j - n(n-2) = \sum_{j=1}^{n-2} nj - n(n-2) = \sum_{j=1}^{n-3} nj + n(n-2) - n(n-2) = n \sum_{j=1}^{n-3} j$$