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Homework number: 6
Homework Title: Exercise 7.10

Problem description:

(a) For a given set of data points, $t_1, ..., t_n$, define the function $\pi(t)$ by

$$\pi(t) = (t - t_1)(t - t_2)...(t - t_n). \tag{1}$$

Show that

$$\pi^{'}(t_j) = (t_j - t_1)...(t_j - t_{j-1})(t_j - t_{j+1})....(t_j - t_n). \tag{2}$$

(b) Use the result of part a to show that the jth Lagrange basis function can be expressed as

$$l_j(t) = \frac{\pi(t)}{(t - t_j)\pi'(t_j)}. \tag{3}$$

Problem solution:

(a) To get the derivation of the function π at t_j I applied the product rule to get the derivation of the function. The data points t_i with i=1..n are constants.
I replaced some parts of the equation to apply the product rule. I simplified the equation

$$\pi(t) = (t - t_1)(t - t_2)...(t - t_n). \tag{4}$$

to

$$\pi(t) = (t - t_1) * a(t) \tag{5}$$

with $a(t) = (t - t_2)...(t - t_n)$.
Than I calculated the derivation with the product rule.

$$\pi^{'}(t) = 1 * a(t) + (t - t_1) * a^{'}(t) \tag{6}$$

I replaced a(t) and introduced a new function $b(t) = (t - t_3)...(t - t_n)$ to replace parts of the equation.
The function a(t) could also be expressed as
$a(t) = (t - t_2) * b(t)$, $b(t) = (t - t_3) * c(t)$ and so on.
The next derivation gives

$$\pi^{'}(t) = 1 * (t - t_2)...(t - t_n) + (t - t_1) * (1 * b(t) + (t - t_2) * b^{'}(t)) \quad (7)$$

I continued with the replacement and derivation of the functions and finally got this result.

$$\begin{gathered}\pi^{'}(t) = (t - t_2)(t - t_3)...(t - t_n) + \\ +(t - t_1)(t - t_3)(t - t_4)...(t - t_n) + \\ +(t - t_1)(t - t_2)(t - t_4)...(t - t_n) + ... \\ +(t - t_1)...(t - t_{n-2})(t - t_n) + \\ +(t - t_1)...(t - t_{n-1}) * 1\end{gathered}$$

Than I evaluated the function at t_j with $1 < j < n$ and found out that all terms with $(t_j - t_j) = 0$ in it, fall off.

$$\begin{gathered}\pi^{'}(t) = (t_j - t_2)(t_j - t_3)... * 0 * ...((t_j - t_n) + ... \\ +(t_j - t_1)(t_j - t_2)(t_j - t_4)...(t_j - t_{j-1})(t_j - t_{j+1})...(t_j - t_n) + 0\end{gathered}$$

Hence I got

$$\pi^{'}(t_j) = (t_j - t_1)...(t_j - t_{j-1})(t_j - t_{j+1})....(t_j - t_n). \quad (8)$$

(b) The formula for the *Langrange* basis function is

$$l_j(t) = \frac{\Pi_{k=1,k\neq j}^{n}(t - t_k)}{\Pi_{k=1,k\neq j}^{n}(t_j - t_k)} \quad (9)$$

This is equal to

$$\frac{(t - t_1)...(t - t_{j-1})(t - t_{j+1})....(t - t_n)}{(t_j - t_1)...(t_j - t_{j-1})(t_j - t_{j+1})....(t_j - t_n)} \quad (10)$$

The numerator of the equation can be expressed with the function π as defined in the upper part

$$(t - t_1)...(t - t_{j-1})(t - t_{j+1})....(t - t_n) = \frac{\pi(t)}{(t - t_j)} \quad (11)$$

The denominator of the formular can be expressed by substitution with the derivation of π evaluated at t_j

$$(t_j - t_1)...(t_j - t_{j-1})(t_j - t_{j+1})....(t_j - t_n) = \pi^{'}(t_j) \tag{12}$$

And hence I got

$$\frac{\pi(t)}{(t - t_j)\pi'(t_j)} = l_j(t) \tag{13}$$