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**Homework Title:** 5.11.

**Problem description:**

Suppose you are using the secant method to find a root  $x^*$  of a nonlinear equation  $f(x) = 0$ . Show that if at any iteration it happens to be the case that either  $x_k = x^*$  or  $x_{k-1} = x^*$  (but not both), then it will also be true that  $x_{k+1} = x^*$ .

**Problem solution:**

In the secant method the next iteration is found with:

$$x_{k+1} = x_k - f(x_k) \frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})}.$$

Case  $x_k$  has already the value of a root  $x^*$ :

$$x_{k+1} = x^* - f(x^*) \frac{x^* - x_{k-1}}{f(x^*) - f(x_{k-1})} = x^* - 0 \cdot \frac{x^* - x_{k-1}}{0 - f(x_{k-1})} = x^*.$$

Case  $x_{k-1}$  has already the value of a root  $x^*$ :

$$x_{k+1} = x_k - f(x_k) \frac{x_k - x^*}{f(x_k) - 0} = x_k - x_k + x^* = x^*.$$

**Results:**

We see that in both cases the iteration will not lose the root  $x^*$  and  $x_{k+1} = x^*$ .