First name: Sebastian
Last name: Gumpold
Date: 25.12.03
Homework number: 5
Homework Title: Excercise 5.3 a), b)

Problem description:

Newton's method is sometimes used to implement the built-in square root function on a computer, with the initial guess supplied by a lookup table.

(a) What is the Newton iteration for computing the square root of a positive number y (i.e., for solving the equation $f(x)=x^2-y=0$, given y)?

(b) If we assume that the starting guess has an accuracy of 4 bits, how many iterations would be necessary to attain 24-bit accuracy? 53-bits accuracy?

Problem solution:

(a) Newton's method uses the following iteration scheme to find the zero of a function f(x): $x_{k+1}=x_k-f(x_k)/f'(x_k)$. So we simply use this method for the given equation $f(x)=x^2-y=0$ with given y . One can easily see that $f'(x)=2x$. Using these values we get $x_{k+1}=x_k-(x_k^2-y)/(2*x_k)$, where y is the value we want to compute the root of.

This formula can be simplified to $x_{k+1}=x_k-\frac{x_k}{2}+\frac{y}{2*x_k}$, so that we finally get this iteration $x_{k+1}=\frac{x_k+y/x_k}{2}$.

(b) As the convergence rate of Newton's method for simple root is quadratic we easily can see the accuracy after every iteration step:

iteration step	correct bits
0	4
1	8
2	16
3	32
4	64

As one can see an accuracy of 24 bit is reached after 3 iterations and and accuracy of 53 bit is reached after 4 iterations.

Results:

(a) The iteration scheme is $x_{k+1}=\frac{x_k+y/x_k}{2}$.

(b) 24-bit accuracy is reached after 3 iterations, 53-bit accuracy after 4 iterations.