

Homework 5

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Problem 5.2: Write out Newton's iteration for solving each of the following nonlinear equations:

(a) $x^3 - 2x - 5 = 0$

(b) $e^{-x} = x$

(c) $x \cdot \sin x = 1$

In general, Newton's iteration for solving an equation of the type $f(x) = 0$ with some chosen starting value x_0 is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)},$$

where the x_n 's are successive approximations for the solution, if the iteration process converges.

ad (a):

Equation to solve:

$$f(x) = x^3 - 2x - 5 = 0$$

Derivative of this function:

$$f'(x) = 3x^2 - 2$$

Iteration formula:

$$x_{n+1} = x_n - \frac{x_n^3 - 2x_n - 5}{3x_n^2 - 2}$$

Starting value:

$$x_0 = 2.5$$

Illustrated iteration process:

Iteration number	x_i	$f(x_i)$	$f'(x_i)$
0	2.5	5.625	16.75
1	2.16418	0.807945	12.051
2	2.09714	0.0288817	11.1939
3	2.09456	0.0000418649	11.1615
4	2.09455	≈ 0	11.1614

ad (b):

Equation to solve:

$$f(x) = e^{-x} - x = 0$$

Derivative of this function:

$$f'(x) = -e^{-x} - 1$$

Iteration formula:

$$x_{n+1} = x_n - \frac{e^{-x_n} - x_n}{-e^{-x_n} - 1}$$

Starting value:

$$x_0 = 1.0$$

Illustrated iteration process:

Iteration number	x_i	$f(x_i)$	$f'(x_i)$
0	1.0	-0.632121	-1.36788
1	0.537883	0.04160005	-1.58398
2	0.566987	0.00024495	-1.56723
3	0.567143	≈ 0	-1.56714

ad (c):

Equation to solve:

$$f(x) = x \cdot \sin x - 1 = 0$$

Derivative of this function:

$$f'(x) = \sin x + x \cdot \cos x$$

Iteration formula:

$$x_{n+1} = x_n - \frac{x_n \cdot \sin x_n - 1}{\sin x_n + x_n \cdot \cos x_n}$$

Starting value:

$$x_0 = 0.5$$

Illustrated iteration process:

Iteration number	x_i	$f(x_i)$	$f'(x_i)$
0	0.5	0.106531	-1.60653
1	0.566311	0.00130451	-1.56762
2	0.567143	≈ 0	-1.56714