First name: Franz-Josef
Last Name: Auernigg
Date: 14.01.04
Homework number: 5
Homework Title: Exercise 5.10

Problem description:

Carry out one iteration of Newton's method applied to the system of nonlinear euquations.

$$x_1^2 - x_2^2 = 0,$$
$$2x_1x_2 = 1,$$

with starting value $x_0 = \begin{vmatrix} 0 \\ 1 \end{vmatrix}$.

Problem solution:

The nonlinear system looks like this:

$$f(x) = \begin{vmatrix} x_1^2 - x_2^2 \\ 2 * x_1 * x_2 \end{vmatrix} = \begin{vmatrix} 0 \\ 1 \end{vmatrix} \quad (1)$$

At first I determined the Jacobian matrix by calculating the partial derivates of the nonlinear system.

$$J_f(x_0) = \begin{vmatrix} 2x_1 & -2x_2 \\ 2x_2 & 2x_1 \end{vmatrix} \quad (2)$$

For x_0 we get $f(x_0) = \begin{vmatrix} -1 \\ 0 \end{vmatrix}$.

For $J_f(x_0)$ we get:

$$J_f(x) = \begin{vmatrix} 0 & -2 \\ 2 & 0 \end{vmatrix} \quad (3)$$

Then I had to solve the system:

$$J_f(x) * s_0 = \begin{vmatrix} 0 & -2 \\ 2 & 0 \end{vmatrix} * s_0 = \begin{vmatrix} -1 \\ 0 \end{vmatrix} = -f(x_0) \tag{4}$$

For s_0 I got the resulting vector: $\begin{vmatrix} -0.5 \\ 0 \end{vmatrix}$.
Now I calculated the x_1 for the next step of the iteration in this way:

$$x_1 = x_0 + s_0 = \begin{vmatrix} -0.5 \\ 1 \end{vmatrix} \tag{5}$$

Results:

The Result from the first step of Newtons Iteration is

$$x_1 = \begin{vmatrix} -0.5 \\ 1 \end{vmatrix} \tag{6}$$