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Homework Title: 4.2.

Problem description:

What are the eigenvalues and corresponding eigenvectors of the following matrix?

$$A = \begin{bmatrix} 1 & 2 & -4 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

Problem solution:

The eigenvalues are the roots λ_i of the characteristic polynomial $\det(A - \lambda I) = 0$ in λ of degree n .

$$\text{Characteristic polynomial: } (1 - \lambda)(2 - \lambda)(3 - \lambda).$$

The eigenvector x_i corresponding to λ_i is the solution of $(A - \lambda_i I)x = 0$.

Results:

The eigenvalues of A : $\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 3$.

The eigenvectors of A:

λ_1 :

$$\begin{bmatrix} 0 & 2 & -4 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2y - 4z \\ y + z \\ 2z \end{bmatrix} =! \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$
$$x = a, y = 0, z = 0. a \in \mathbf{R}.$$

λ_2 :

$$\begin{bmatrix} -1 & 2 & -4 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -x + 2y - 4z \\ z \\ z \end{bmatrix} =! \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$
$$x = b, y = \frac{b}{2}, z = 0. b \in \mathbf{R}.$$

λ_3 :

$$\begin{bmatrix} -2 & 2 & -4 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2x + 2y - 4z \\ -y + z \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$x = -c, \quad y = c, \quad z = c. \quad c \in \mathbf{R}.$$