Homework 4

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Exercise 4.3: Let

$$\mathbf{A} = \begin{pmatrix} 1 & 4 \\ 1 & 1 \end{pmatrix}.$$

Your answers to the following questions should be numeric and specific to this particular matrix, not just the general definition.

ad c): What are the eigenvalues of $\mathbf{A}$?

Solution: The Eigenvalues of $\mathbf{A} = \begin{pmatrix} 1 & 4 \\ 1 & 1 \end{pmatrix}$ are the solutions of

$$det(\mathbf{A} - \lambda\mathbf{I}) = det(\begin{pmatrix} 1-\lambda & 4 \\ 1 & 1-\lambda \end{pmatrix}) = (1-\lambda)^2 - 4 = \lambda^2 - 2\lambda - 3 = 0.$$

These solutions are given by

$$\lambda_{1,2} = 1 \pm \sqrt{1+3} = 1 \pm 2.$$

So the first eigenvalue is $\lambda_1 = 3$ and the second is $\lambda_2 = -1$.

ad f): To what eigenvector of $\mathbf{A}$ will power iteration ultimately converge?

Solution: Power iteration converges to the eigenvector corresponding to the unique eigenvalue of maximum modulus; in our case this is $\lambda_1 = 3$. The corresponding eigenvector $\mathbf{v}_1 := \begin{pmatrix} x \\ y \end{pmatrix}$ to λ_1 is given by the solution of

$$\begin{pmatrix} 1 & 4 \\ 1 & 1 \end{pmatrix} \mathbf{v}_1 = \lambda_1 \mathbf{v}_1$$

what is equivalent to the following system of equations:

$$\begin{aligned} x + 4y &= 3x \\ x + y &= 3y \end{aligned}$$

That means $x = 2y$, so power iteration will converge to an eigenvector of the form $\mathbf{v}_1 = \begin{pmatrix} y \\ 2y \end{pmatrix}$ with an arbitrary real y.

ad i): What eigenvalue of **A** would be obtained if inverse iteration were used with shift $\sigma = 2$?

Solution: Inverse iteration with shift $\sigma := 2$ converges to the eigenvalue λ which is closest to σ. In our case this would also be $\lambda_1 = 3$ since

$$|\lambda_1 - \sigma| = 1 < 3 = |\lambda_2 - \sigma|$$

holds. That means, λ_1 is closer to σ than λ_2.

ad j): If **QR** iteration were applied to **A**, to what form would it converge: diagonal or triangular? Why?

Solution: **QR** iteration does not converge to diagonal form, since **A** is not symmetric, but it converges to triangular form because the eigenvalues $\lambda_1 = 3$ and $\lambda_2 = -1$ are distinct in modulus.