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Date: 11-02-2004
Homework number: 4
Homework title: Exercise 4.31

Problem Description

(a) If λ is an eigenvalue of an orthogonal matrix $\mathbf{Q}$, show that $|\lambda| = 1$.

(b) What are the singular values of an orthogonal matrix?

Problem Solution

Let λ be the an eigenvalue of the orthogonal matrix $\mathbf{Q}$. So the following relation holds: $\mathbf{Q}x = \lambda x$. Taking the complex transpose yields to $x^*\mathbf{Q}^* = \bar{\lambda}x^*$. As $\mathbf{Q}$ is a real matrix $\mathbf{Q}^* = \mathbf{Q}^T$ we get the following equation

$$x^*x = x^*\mathbf{Q}^*\mathbf{Q}x = \bar{\lambda}\lambda x^*x$$

By definition the eigenvector x is non-zero leading to $|\lambda| = 1$.

The singular values of $\mathbf{Q}$ are the nonnegative square roots of the eigenvalues of $\mathbf{Q}^T\mathbf{Q} = \mathbf{I}$. The eigenvalues of $\mathbf{I}$ are all 1 so the singular values of an orthogonal matrix are all 1 too.