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Homework title: Exercise 3.26

Problem Description

Let $c = \cos(\theta)$ and $s = \sin(\theta)$ for some angle θ. Give a detailed geometric description of the effects on vectors in the Euclidean plane $\mathbb{R}^2$ of each the following 2×2 orthogonal matrices.

$$G = \begin{bmatrix} c & s \\ -s & c \end{bmatrix} \qquad H = \begin{bmatrix} -c & s \\ s & c \end{bmatrix}$$

Problem Solution

Multiplication of any vector by G returns an about the origin by θ clockwise rotated vector ...

$$G \begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{pmatrix} r\cos\alpha \\ r\sin\alpha \end{pmatrix} = \\ = \begin{pmatrix} r\cos\alpha\cos\theta + r\sin\alpha\sin\theta \\ -r\cos\alpha\sin\theta + r\sin\alpha\cos\theta \end{pmatrix} = \begin{pmatrix} r\cos(\alpha-\theta) \\ r\sin(\alpha-\theta) \end{pmatrix}$$

Multiplication of any vector by H returns an about the origin by θ counterclockwise rotated vector additionally reflected by the y-axis ...

$$H \begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{pmatrix} r\cos\alpha \\ r\sin\alpha \end{pmatrix} = \\ = \begin{pmatrix} -(r\cos\alpha\cos\theta - r\sin\alpha\sin\theta) \\ r\cos\alpha\sin\theta + r\sin\alpha\cos\theta \end{pmatrix} = \begin{pmatrix} -r\cos(\alpha+\theta) \\ r\sin(\alpha+\theta) \end{pmatrix}$$