First Name: Annemarie
Last Name: Mayer
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Homework Number: 3
Homework Title: Exercise 3.17

Problem Description:

Determine the Householder transformation that annihilates all but the first entry of the vector $[1\ 1\ 1\ 1]^T$. Specifically, if

$$\left(I - 2\frac{vv^T}{v^Tv}\right)\begin{bmatrix}1\\1\\1\\1\end{bmatrix} = \begin{bmatrix}\alpha\\0\\0\\0\end{bmatrix},$$

what are the values of α and v?

Problem Solution:

Let

$$\mathbf{H} = I - 2\frac{vv^T}{v^Tv} \quad \text{where} \quad v = \mathbf{a} - \alpha \cdot e_1 \quad \text{and} \quad \alpha = \pm||\mathbf{a}||_2.$$

Then

$$\mathbf{Ha} = \begin{bmatrix}\alpha\\0\\\vdots\\0\end{bmatrix}$$

holds for the Householder matrix $\mathbf{H}$ and any vector $\mathbf{a}$ of the same dimension.

Results:

In this example, for $\mathbf{a} = [1\ 1\ 1\ 1]^T$, we get

$$\alpha = \pm 2.$$

To avoid cancellation, we choose $\alpha = -2$ and receive

$$v = [3\ 1\ 1\ 1]^T$$

$$\mathbf{H} = \begin{bmatrix}-1/2 & -1/2 & -1/2 & -1/2\\-1/2 & 5/6 & -1/6 & -1/6\\-1/2 & -1/6 & 5/6 & -1/6\\-1/2 & -1/6 & -1/6 & 5/6\end{bmatrix}$$