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**Homework Title:** Excercise 3.18

**Problem description:**

Suppose that you are computing the QR factorization of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \end{bmatrix}$$

by Householder transformations.

(a) How many Householder transformations are required?
(b) What does the first column of A become as a result of applying the first Householder transformation?
(c) What does the first column then become as a result of applying the second Householder transformation?
(d) How many Givens rotations would be required to compute the QR factorization of A?

**Problem solution:**

(a) 3 transformations are needed here, because the ith householder transformation sets all values in the ith column that are below the diagonal to zero. And as the last colums values below the diagonal are not intitially zero we have to apply one householder transformation for every column.

(b) The first column becomes $\begin{bmatrix} -2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ .

To get this result, we simply take the first column of matrix A – we call it vector a. The Euclidean norm of vector a becomes the first element of the first column of $H_1 A$ . The rest (all values below the diagonal) of the new matrix $H_1 A$ are set to zero. To get the right sign for the norm we take a look at the computation of vector $v_1$ , which is needed to compute $H_1$ :

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} -2 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \\ 1 \end{bmatrix} .$$

As you can see here we multiply the Euclidean norm of vector a (that is 2) by -1 to avoid cancellation.

(c) As the ith householder transformation only effects the columns i to n of an mxn matrix it is applied to, all columns that are smaller than i stay the same.

(d) For the given 4x3 matrix for every of the 3 columns 4-col.number rotations are needed. In sum this are 3 + 2 + 1 = 6 needed Givens rotations.

**Results:**

(a) 3 transformation matrices are needed here.

(b) The first column becomes $\begin{bmatrix} -2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ .

(c) The first column of the matrix stays $\begin{bmatrix} -2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ .

(d) 6 Givens rotations are needed here.

**Discussion and Comments:**

One can see in the given example that more Givens rotations than householder transformations are needed. But one has to notice that the Givens rotation therefore only effect 2 rows and 2 columns of the matrix whereas the ith householder transformation effects the whole matrix (except the columns smaller that i). So QR factorization with Givens rotations need not to be much worse than with householder transformation.