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Homework Title: Exercise 3.16

Problem description:

Consider the vector a as an $n \times 1$ matrix.

(a) Write out its QR factorization, showing the matrices Q and R explicitly.

(b) What is the solution to the linear least squares problem $ax \cong b$ where b is a given n-vector

Problem solution:

if $A \in \mathbb{R}^{m \times n}$ with $m \geq n$ and $rank\ (A) = n$ then it can be factored as

$$A = QR$$

- $R \in \mathbb{R}^{n \times n}$ is upper triangular wirh $r_{ii} > 0$
- $Q \in \mathbb{R}^{m \times n}$ satisfies $Q^T Q = I$ (Q is an orthogonal matrix)

(a) Given vector a is a $n \times 1$ matrix. When using Gram-Schmidt method for this special case we get:

$A = [a_1] \quad Q = [q_1] \quad R = r_{11}$

$$A = a = \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{n1} \end{pmatrix} \qquad R = r_{11} = ||a|| = \sqrt{\sum_{i=1}^{n} a_{i1}^2} \qquad Q = q_1 = \frac{a}{||a||} = \begin{pmatrix} \frac{a_{11}}{||a||} \\ \frac{a_{21}}{||a||} \\ \vdots \\ \frac{a_{n1}}{||a||} \end{pmatrix}$$

(b) Therefore the solution to the linear least squares problem can be determined by:

$$q_1^T a x = Rx = r_{11} x = ||a|| x = q_1^T b$$
$$\Rightarrow x = \frac{q_1^T b}{||a||}$$