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Date: 14.12.03
Homework number: 3
Homework Title: Exercise 3.19

Problem description:

Consider the vector:

$$a = \begin{vmatrix} 2 \\ 3 \\ 4 \end{vmatrix} \tag{1}$$

(a) Specify the elementary elemination matrix that annihilates the third component of a.
(b) Specify the Householder transformation that annihilates the third component of a.
(c) Specify a Givens rotation that annihilates the third component of a.
(d) When annihilating a given nonzero component of any vector, is it ever possible for the corresponding elementary elimination matrix and Householder transformation to be the same? Why?
(e) When annihilating a given nonzero component of any vector, is it ever possible for a corresponding Householder transformation and Givens rotation to be the same? Why?

Problem solution:

(a) 'a' has to be transformed to the vector on the right side by multiplying it with the elementary elimination matrix.

$$E * a = \begin{vmatrix} 2 \\ 3 \\ 0 \end{vmatrix} \tag{2}$$

The elementary elimination matrix must be non-singular.

(b) The equation for the Householder transformation matrix is

$$H = I - 2 * \frac{v * v^T}{v^T * v}, \tag{3}$$

$$H * a = a - 2 * \frac{v^T * a}{v^T * v} * v, \tag{4}$$

The Vector v is calculated from the vector a, α and unit vectors.

$$v = a - \alpha * e_1 \tag{5}$$

$$\alpha = \pm \|a\|_2 \tag{6}$$

To eliminate the third component of 'a' I left the first component untouched and handled the vector 'a' like a vector with two dimensions. Then I annihilated the last component.

(c) For the general case the following equation is used for the Givens Rotation.

$$\begin{vmatrix} c & s \\ -s & c \end{vmatrix} \tag{7}$$

$$c...cosine, c = \frac{a_1}{\sqrt{a_1^2 + a_3^2}} = \frac{2}{\sqrt{20}}$$
$$s...sine, s = \frac{a_3}{\sqrt{a_1^2 + a_3^2}} = \frac{4}{\sqrt{20}}$$

Results:

(a)The elementary elimination matrix is

$$E = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -4/3 & 1 \end{vmatrix} \tag{8}$$

The Determinant of this matrix is 1. Therefore it's non-singular

(b)

$$a = \begin{vmatrix} 2 \\ 3 \\ 4 \end{vmatrix} \tag{9}$$

$$a2 = \begin{vmatrix} 3 \\ 4 \end{vmatrix} \tag{10}$$

$$\alpha = \pm\sqrt{3^2 + 4^2} = \pm 5 \tag{11}$$

The sign for α is choosen negative to avoid cancellation of 'a' and $\alpha * e_1$.

$$v = \begin{vmatrix} 3 \\ 4 \end{vmatrix} - \alpha * \begin{vmatrix} 1 \\ 0 \end{vmatrix} = \begin{vmatrix} 3+5 \\ 4 \end{vmatrix} \tag{12}$$

$$H1 = I - 2 * \frac{v * v}{vv} = \begin{vmatrix} -.6 & -.8 \\ -.8 & .6 \end{vmatrix} \tag{13}$$

$$H1 * a1 = [-50]. \tag{14}$$

Adding the first component I got the following vector:

$$v = \begin{vmatrix} 2.0000 \\ -5.0000 \\ 0.0000 \end{vmatrix} \tag{15}$$

(c)

$$G = \begin{vmatrix} c & 0 & s \\ 0 & 1 & 0 \\ -s & 0 & c \end{vmatrix} = \begin{vmatrix} 0.4472 & 0 & 0.8944 \\ 0 & 1.0000 & 0 \\ -0.8944 & 0 & 0.4472 \end{vmatrix} \tag{16}$$

$$G * \begin{vmatrix} 2 \\ 3 \\ 4 \end{vmatrix} = \begin{vmatrix} 4.4721 \\ 3.0000 \\ 0 \end{vmatrix} \tag{17}$$

(d) It is not possible, because the element M(1,1) of the elementary elimination matrix is always 1. To have the element of H(1,1) of the Householder transfomation =1 the element v(1) must be 1. By definition, this would be possible only if a(1) equals to the norm(a)+1, what is obviously not possible, because no vector component can be greater than the norm of this vector.

(e) Householder and Givens Rotation can be the same. For example Householder is $H1 = \begin{vmatrix} -0.8 & -0.6 \\ -0.6 & 0.8 \end{vmatrix}$ and Givens is $G1 = \begin{vmatrix} -0.8 & -0.6 \\ -0.6 & 0.8 \end{vmatrix}$.
Both lead when applied to $\begin{vmatrix} 4 \\ 3 \end{vmatrix}$ to $\begin{vmatrix} -5 \\ 0 \end{vmatrix}$.

Discussion and Comments (optional):

(b) To confirm the results, I calculated the norm of this vector:

$$\|v\|_2 = \sqrt{29} \tag{18}$$

This is the same like the norm of v without the Transformation.