Homework 2

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Exercise 2.18: Prove that the matrix

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

has no **LU** factorization, i.e., no lower triangular matrix **L** and upper triangular matrix **U** exist such that $\mathbf{A} = \mathbf{LU}$.

Solution:

Assume that there are matrices

$$\mathbf{L} = \begin{pmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{pmatrix}$$

and

$$\mathbf{U} = \begin{pmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{pmatrix}$$

such that $\mathbf{A} = \mathbf{LU}$. Then we get

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} l_{11}u_{11} & l_{11}u_{12} \\ l_{21}u_{11} & l_{21}u_{12} + l_{22}u_{22} \end{pmatrix} = \begin{pmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{pmatrix} \begin{pmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{pmatrix} = \mathbf{LU}$$

That means we have four equations:

1. $0 = l_{11}u_{11} \Longrightarrow l_{11} = 0 \vee u_{11} = 0$
2. $1 = l_{11}u_{12} \Longrightarrow l_{11} \neq 0 \wedge u_{12} \neq 0 \overset{1.}{\Longrightarrow} u_{11} = 0$
3. $1 = l_{21}u_{11} \Longrightarrow l_{21} \neq 0 \wedge u_{11} \neq 0 \Longrightarrow Contradiction!$
4. $\ldots$

This disproves our assumption.