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Homework Number: 2
Homework Title: Exercise 2.7

Problem Description:

Let

$$\mathbf{A} = \begin{bmatrix} 1 & 1+\epsilon \\ 1-\epsilon & 1 \end{bmatrix}.$$

a) What is the determinant of **A**?

b) In floating-point arithmetic, for what range of values of ϵ will the computed value of the determinant be zero?

c) What is the LU factorization of **A**?

d) In floating-point arithmetic, for what range of values of ϵ will the computed value of U be singular?

Problem Solution:

a) The determinant of **A**:

$$\begin{aligned} det(\mathbf{A}) &= (1 \cdot 1) - (1+\epsilon)(1-\epsilon) \\ &= 1 - (1-\epsilon^2) \\ &= \epsilon^2 \end{aligned}$$

b) *Rounding by chopping:*
$\epsilon \in \left[0, \ldots, \sqrt{UFL}\right)$ as ϵ^2 must be less than UFL.
Rounding to nearest:
ϵ^2 must not be rounded up to UFL, therefore $\epsilon \in \left[0, \ldots, \sqrt{\frac{UFL}{2}}\right)$.

c) Eliminating element $\mathbf{A}(2,1)$ with pivot 1 gives

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -\frac{1-\epsilon}{1} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \epsilon - 1 & 1 \end{bmatrix}$$

The inverse of **M** gives **L**.

$$\begin{bmatrix} 1 & 0 \\ \epsilon - 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{array}{lll} 1 \cdot a = 1 & \Rightarrow & a = 1 \\ 1 \cdot b = 0 & \Rightarrow & b = 0 \\ (\epsilon - 1)a + c = 0 & \Rightarrow & c = 1 - \epsilon \\ (\epsilon - 1)b + d = 1 & \Rightarrow & d = 1 \end{array}$$

$$\mathbf{L} = \begin{bmatrix} 1 & 0 \\ 1 - \epsilon & 1 \end{bmatrix}$$

U is defined as **M** by **A**.

$$\mathbf{U} = \begin{bmatrix} 1 & 0 \\ \epsilon - 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 + \epsilon \\ 1 - \epsilon & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 + \epsilon \\ 0 & \epsilon^2 \end{bmatrix}$$

d) **U** singular $\Leftrightarrow det(\mathbf{U}) = 0$.
$det(\mathbf{U}) = 1 \cdot \epsilon^2 - 0 = \epsilon^2$.
The solution equals to the one in item b).

Results:

a) $det(\mathbf{A}) = \epsilon^2$

b) Rounding by chopping: $\epsilon \in \left[0, \ldots, \sqrt{UFL}\right)$
Rounding to nearest: $\epsilon \in \left[0, \ldots, \sqrt{\frac{UFL}{2}}\right)$

c) $\mathbf{L} = \begin{bmatrix} 1 & 0 \\ 1 - \epsilon & 1 \end{bmatrix}, \quad \mathbf{U} = \begin{bmatrix} 1 & 1 + \epsilon \\ 0 & \epsilon^2 \end{bmatrix}$

d) see b).