First name: Sebastian
Last name: Gumpold
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Homework number: 2
Homework Title: Excercise 2.8

Problem description:

Let A and B be any two n x n matrices.
(a) Prove that $(AB)^T = B^T A^T$
(b) If A and B are both nonsingular, prove that $(AB)^{-1} = B^{-1} A^{-1}$

Problem solution:

(a) For this proof it has to be shown that each element of the matrix $(AB)^T$ is the same as in $B^T A^T$.
(b) It has to be shown, that $B^{-1} A^{-1}$ is the left and right inverse of (AB).

Results:

(a) The following notation is used in the following proof: a_{ij}^T is the (i,j)th element of A^T, so that $a_{ij}^T = a_{ji}$.

Proof:

The (i,j)th element of AB is $\sum_{k=1}^{n} a_{ik} b_{kj}$, so the (i,j)th element of $(AB)^T$ is $\sum_{k=1}^{n} a_{jk} b_{ki}$. This is evident, because the (i,j)th element of $(AB)^T$ is the (j,i)th element of AB, which is the scalar product of the jth row of A and the ith column of B.

And further we can make the following equations:

$\sum_{k=1}^{n} a_{jk} b_{ki} = \sum_{k=1}^{n} a_{kj}^T b_{ik}^T = \sum_{k=1}^{n} b_{ik}^T a_{kj}^T$ which is the (i,j)th element of $B^T A^T$. ■

(b) The inverse of A and B exist, because both matrices are nonsingular.

Proof:

$(B^{-1} A^{-1})(AB) = B^{-1}(A^{-1} A) B = B^{-1} B = I$ and

$(AB)(B^{-1} A^{-1}) = A(B B^{-1}) A^{-1} = A A^{-1} = I$. ■

Discussion and Comments:

none.