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Homework number: 2
Homework Title: Exercise 2.4

Problem description:

(a) Show that the following matrix is singular.
Consider the system

$$\begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 1 & 3 & 2 \end{vmatrix} \tag{1}$$

(b) If b = [2 4 6] ^{T} , how many solutions are there to the system Ax = b?

Problem solution:

(a) A matrix is singular if it is not non-singular.
A matrix(n x n) is non-singular, if it has any of the following equivalent properties:

- inverse of A, denoted by A^{-1} exists
- det(A) $\neq 0$
- rank(A) = n
- For any vector z $\neq o, Az \neq o$

So i chose to check the rank, the inverse and the determinant of the matrix with Matlab.
(b) I tried to solve the system in matlab with the command: x=A\b

Results:

(a) The Rank: rank(A)=2 != n=3)
The Determinant: det(A) of the matrix is 0.
Inv(A) gives 'Inf' Solutions. That means that there exists no inverse of the matrix.
That means that the Matrix A is singular.

(b) x=A\b gives infinite solutions.
The Linear System A*x=b is represented by the matrix:

$$\begin{vmatrix} 1 & 1 & 0 & 2 \\ 1 & 2 & 1 & 4 \\ 1 & 3 & 2 & 6 \end{vmatrix} \tag{2}$$

I solved this linear System with the Gauss-Algorithm (in Matlab: S=rref(A)) and got:

$$\begin{vmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{vmatrix} \tag{3}$$

The Solution of the System can be described by the line:

$$x = \begin{vmatrix} 0 \\ 2 \\ 0 \end{vmatrix} \pm r * \begin{vmatrix} 1 \\ -1 \\ 1 \end{vmatrix} \tag{4}$$

Discussion and Comments:

r can be any real number. There are infinite real numbers and thats why we have infinite solutions.