

# Homework 1

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**Exercise 1.5:** Consider the function  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  defined by  $f(x, y) = x - y$ . Measuring the size of the input  $(x, y)$  by  $|x| + |y|$ , and assuming that  $|x| + |y| \approx 1$  and  $x - y \approx \epsilon$ , show that  $\text{cond}(f) \approx \frac{1}{\epsilon}$ . What can you conclude about the sensitivity of subtraction?

*Solution:*

$$f(x \pm \Delta x, y \pm \Delta y) = x - y \mp (\Delta x + \Delta y)$$

$$f(x \pm \Delta x, y \pm \Delta y) - f(x, y) = \mp(\Delta x + \Delta y)$$

When inserted in the definition of the condition number this yields:

$$\text{cond}(f) = \frac{\frac{|\mp(\Delta x + \Delta y)|}{\epsilon}}{\frac{|\Delta x + \Delta y|}{|x| + |y|}} = \frac{1}{\epsilon}.$$

That means that subtraction is ill-conditioned if  $x - y$  is small.