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***Problem 1.11:*** *If* $x \approx y$*, then we would expect some cancellation in computing* $\log x - \log y$*. On the other hand,* $\log x - \log y = \log \frac{x}{y}$*, and the latter involves no cancellation. Does this mean that computing* $\log \frac{x}{y}$ *is likely to give a better result? (Hint: For what value is the log-function sensitive?)*

**Solution:**

If $x \approx y$, we also have $\ln x \approx \ln y$ (as ln is continuous), and therefore it is evident that computing $\log x - \log y$ will lead to some cancellation.
For the function $f(x) := \log x$ we may compute the condition number as follows:

$$\text{cond} \approx \left| \frac{x\, f'(x)}{f(x)} \right| = \left| \frac{x\, \frac{1}{x}}{\ln x} \right| = \left| \frac{1}{\ln x} \right|$$

Thus, the condition number for log may become extremely high if $\log x$ is almost zero, which means $x \approx 1$.
So we see: Whenever $\log x - \log y$ yields a "big" problem in terms of cancellation, the alternative computation $\log \frac{x}{y}$ will be extremely bad conditioned.
Therefore, we may conclude that computing $\log \frac{x}{y}$ instead of $\log x - \log y$ is not likely to give a (significantly) better result, as only the type of the expected computation error changes, but the problem itself will remain.