First name: Franz-Josef
Last Name: Auernigg
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Homework number: 1
Homework Title: Exercise 1.9

Problem description:

(a) Compute the surface of the Earth, with r=6370km (using 4 digit decimal arithmetic)
(b) Using the same formula and precision, compute the difference in surface area if the value for the radius is increased by 1km.
(c) Since dA/dr = $8 * \pi * r$, the change in surface area is approximated by $8 * \pi * r * h$ where h is the change in radius. Use this formula, still with four digit arithmetic, to compute the difference in surface area due to an increase of 1 km in radius. How does the value obtained using this approximate formula compare with that obtained from the "exact" formula in part b?
(d) Determine which of the previous two answers is more nearly correct by repeating both computations using higher precision, say, six-digit decimal arithmetic.
(e) Explain the results you obtained in parts a-d.
(f) Try this problem on a computer. How small must the change h in radius be for the same phenomenon to occur? Try both single and double precision, if available.

Problem solution:

(a) The formula for the surface is S= $4 * \pi * r^2$. I created a function S(r) of this formula in Matlab.
The number of arithmetic digits in Matlab can be set with the 'digits(d)'. All calculations should be enclosed by the function vpa(...) (variable-precision-arithmetic). Another way is to round the value with the Matlab function num2str(value,digits).
(b) Calculate the area for r+1 with the formula S and calculate the difference. diff = $S_{r+1} - S_r$
(c) I defined a function approx = inline('8*pi*r*h');
(d) Set the number of digits to 6 with the command 'digits(6)' and repeat the computations from (b) and (c).
(e) See Discussion and Comments.

(f) I used the 'format short' or 'format long' to set the precision in Matlab and defined a function Sdif for the difference between S_{6370+h}, S_h .

Results:

(a) The exact value is 509904363.7818 km^2 .
vpa(S(6370)) = 0.5099e9 = 509900000 km^2 .

(b) vpa(S(6371)) = 0.5101e9
diff = vpa(0.5101e9 - 0.5099e9) = 200000 $km^{2.}$
exact value = $S_{6371} - S_{6370}$ = 5.1006e+008 - 509904363.7818 = 1.5564e+005

(c) vpa(approx(6370,1)) = 0.1601e6
0.1601e6 - 200000 = -39900

(d) vpa(approx(6370,1)) = 160095
vpa(S(6371) - S(6370)) = vpa(0.510064e9 - 0.509904e9) = 160108
exact value = 1.5564e+005
$absoluteError_{approx} = 160095 - 1.5564e + 005 = 4455$
$absoluteError_{S2-S1} = 160108 - 1.5564e + 005 = 4468$

(f) I tried a few values and found out that:
Short format: For r=6370 and h= 0.1, 0.7, 0.8 the approximation function is equal to the normal formula
Long format: i didn't found a result for that but for h=0.01 the results are nearly equal.

Discussion and Comments:

(e) Using the 4-digit decimal arithmetic the result of the function in (c) approximating the difference in surface area is closer to the exact value as the values without approximation (in 4-digit arithmetic).
But using 6-digit decimal arithmetic the approx-function is nearly equal to the result without a approximation in 6-digit arithmetic. I think the reason for that phenomeon is that the approx-function is numerical more stable in 4-digit arithmetic than the computation with the normal formula.
So if we have a arithmetic with more than 6 digits we will use the normal formula, if we have less we will use the approximating function.