

## REVIEW - CHAPTER 5

- 5.1. True or false: A small residual  $\|f(x)\|$  guarantees an accurate solution of a system of nonlinear equations  $f(x)=0$ .
- 5.2. True or false: Newton's method is an example of a fixed-point iteration scheme.
- 5.3. True or false: If an iterative method for solving a nonlinear equation gains more than one bit of accuracy per iteration, then it is said to have a superlinear convergence rate.

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- 5.5. Suppose you are using an iterative method to solve a nonlinear equation  $f(x) = 0$  for a root that is ill-conditioned, and you need to choose a convergence test. Would it be better to terminate the iteration when you find an iterate  $x_k$  for which  $\|f(x_k)\|$  is small, or when  $\|x_k - x_{k-1}\|$  is small? Why?
- 5.6. (a) What is meant by a bracket for a nonlinear function in one dimension?(b) What does this concept have to do with zero finding?
- 5.7. For root finding problems, why must we use an absolute rather than a relative condition number in assessing sensitivity?

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- 5.9. If the errors at successive iterations of an iterative method are as follows, how would you characterize the convergence rate?
  - (a)  $10^{-2}, 10^{-4}, 10^{-8}, 10^{-16}, \dots$
  - (b)  $10^{-2}, 10^{-4}, 10^{-6}, 10^{-8}, \dots$
- 5.12. Suppose you are using the bisection method to find a zero of a nonlinear function, starting with an initial bracketing interval  $[a, b]$ . Give a general expression for the number of iterations that will be required to achieve an error tolerance of  $tol$  for the length of the final bracketing interval.

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- 5.13. What is meant by a *quadratic* convergence rate for an iterative method?
- 5.14. If an iterative method squares the error every two iterations, what is its convergence rate  $r$ ?
- 5.15. (a) What does it mean for a root of an equation to be a *multiple* root?
  - (b) What is the effect of a multiple root on the convergence rate of the bisection method?
  - (c) What is the effect of a multiple root on the convergence rate of Newton's method?

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- 5.17. What is the convergence rate for Newton's method for finding the root  $x = 2$  of each of the following equations? (a)  $f(x) = (x - 1)(x - 2)^2 = 0$ ; (b)  $f(x) = (x - 1)^2(x - 2) = 0$
- 5.20. Let  $g: \mathbf{R} \rightarrow \mathbf{R}$  be a smooth function having a fixed point  $x^*$ . (a) What condition determines whether the iteration scheme  $x_{k+1} = g(x_k)$  is locally convergent to  $x^*$ ? (b) What is the convergence rate? (c) What additional condition implies that the convergence rate is quadratic? (d) Is Newton's method for finding a zero of a smooth function  $f: \mathbf{R} \rightarrow \mathbf{R}$  an example of such a fixed-point iteration scheme? If so, what is the function  $g$  in this case? If not, then explain why not.

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- 5.26. For solving a one-dimensional nonlinear equation, how many function or derivative evaluations are required per iteration of each of the following methods? (a) Newton's method (b) Secant method
- 5.27. Rank the following methods 1 through 3, from slowest convergence rate to fastest convergence rate, for finding a simple root of a nonlinear equation in one dimension: (a) Bisection method (b) Newton's method (c) Secant method
- 5.28. In solving a nonlinear equation in one dimension, how many bits of accuracy are gained per iteration of (a) Bisection method? (b) Newton's method?

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- 5.30. What is meant by inverse interpolation? Why is it useful for root finding problems in one dimension?
- 5.31. Suppose that you are using fixed-point iteration based on the fixed-point problem  $x=g(x)$  to find a solution  $x^*$  to a nonlinear equation  $f(x) = 0$ . Which would be more favorable for the convergence rate: a horizontal tangent of  $g$  at  $x^*$  or a horizontal tangent of  $f$  at  $x^*$ ? Why?
- 5.33. For what type of function is linear fractional interpolation a particularly good choice of zero finder?

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- 5.35. State at least one method for finding all the zeros of a polynomial, and discuss its advantages and disadvantages.
- 5.37. For solving an  $n$ -dimensional nonlinear equation, how many scalar function evaluations are required per iteration of Newton's method?
- 5.39. Give two reasons why secant updating methods for solving systems of nonlinear equations are often more efficient than Newton's method despite converging more slowly.