## REVIEW - CHAPTER 2/2

**True or false:**

- If a matrix $A$ is nonsingular, then the number of solutions to the linear system $Ax = b$ depends on the particular choice of right-hand-side vector $b$,
- An underdetermined system of linear equations $Ax = b$ where $A$ is an $m \times n$ matrix with $m < n$, always has a solution,
- Gaussian elimination without pivoting fails only when the matrix is ill-conditioned or singular,
- In solving a nonsingular system of linear equations, Gaussian elimination with partial pivoting usually yields a small residual even if the matrix is ill-conditioned.



## REVIEW - CHAPTER 2/2

- Specify an elementary elimination matrix that zeros the last two components of the vector: [3 2 -1 4]'.
- Specify a 4 x 4 permutation matrix that interchanges the 2nd and 4th components of any 4-vector.
- Consider the following matrix:

A = [4 -8 1;6 5 7;0 -10 -3], whose LU factorization we wish to compute using Gaussian elimination: What will the initial pivot element be if:

(a) No pivoting is used?

(b) Partial pivoting is used?

(c) Complete pivoting is used?



## REVIEW - CHAPTER 2/2

- If $A$ and $B$ are $n \times n$ matrices, with $A$ nonsingular, and $c$ is an $n$-vector, how would you efficiently compute the product $A^{-1}Bc$?

- In a floating-point system having 10 decimal digits of precision, if Gaussian elimination with partial pivoting is used to solve a linear system whose matrix has a condition number of $10^3$, and whose input data are accurate to full machine precision, about how many digits of accuracy would you expect in the solution?



## REVIEW - CHAPTER 2/2

- List three advantages of Cholesky factorization compared with LU factorization.
- What is the Cholesky factorization of the following matrix: [4 2;2 2] ?
- Suppose you have already solved the $n \times n$ linear system $Ax = b$ by LU factorization and back-substitution. What is the further cost (order of magnitude) of solving a new system:

(a) With the same matrix $A$ but a different right-hand-side vector?

(b) With the matrix changed by adding a matrix of rank one?

(c) With the matrix $A$ changed completely?



## REVIEW - CHAPTER 3

**True or false:**

- A linear least squares problem always has a solution,
- Fitting a straight line to a set of data points is a linear least squares problem, whereas fitting a quadratic polynomial to the data is a nonlinear least squares problem,
- At the solution to a linear least squares problem $Ax \approx b$, the residual vector $r = b - Ax$ is orthogonal to span(A),
- Methods for solving linear least square based on orthogonal factorization are more computationally expensive than the normal equations.



## REVIEW - CHAPTER 3

- In a data-fitting problem in which $m$ data points *(ti, yi)* are fit by a model function $f(t,x)$, where t is the independent variable and x is an n-vector of parameters to be determined, what does it mean for the function $f$ to be *linear* in the components of x?
- Give an example of a linear and nonlinear model function $f(t,x)$.
- In an overdetermined linear least squares problem with model function $f(t,x) = x_1 f_1(t) + x_2 f_2(t) + x_3 f_3(t)$, what will be the rank of the resulting least squares matrix A if we take $f_1(t)=1$, $f_2(t)= t$, and $f_3(t)=1-t$?

## REVIEW - CHAPTER 3

- What is the system of normal equations for the linear least squares problem $Ax \approx b$?
- Why are orthogonal transformations, such as Householder or Givens, often used to solve least squares problems?
- Why are such methods not often used to solve square linear systems?
- Do orthogonal transformations have any advantage over Gaussian elimination for solving square linear systems? If so, state one.



## REVIEW - CHAPTER 3

- Which of the following properties does an $n \times n$ orthogonal matrix necessarily have?
    - (a) It is nonsingular.
    - (b) It preserves the Euclidean vector norm.
    - (c) Its transpose is its inverse.
    - (d) Its columns are orthonormal.
    - (e) It is symmetric.
    - (f ) It is diagonal.
    - (g) Its Euclidean matrix norm is 1.
    - (h) Its Euclidean condition number 1.
- Show that multiplication by an orthogonal matrix preserves the Euclidean norm of a vector.



## REVIEW - CHAPTER 3

- List one advantage and one disadvantage of Givens rotations for QR factorization compared with Householder transformations.
- When used to annihilate the second component of a 2-vector, does a Householder transformation always give the same result as a Givens rotation?
- Compared to the classical Gram-Schmidt procedure, which of the following are advantages of modified Gram-Schmidt orthogonalization?
    - (a)Requires less storage
    - (b) Requires less work
    - (c) Is more stable numerically.



## REVIEW - CHAPTER 3

- In terms of the condition number of the matrix *A* compare the range of applicability of the normal equations method and the Householder QR method for solving the linear least squares problem $Ax \approx b$ [i.e., for what values of cond(*A*) can each method be expected to break down?].

- Let *A* be an $m \times n$ matrix:

(a) What is the maximum number of nonzero singular values that *A* can have?

(b) If rank$(A) = k$, how many nonzero singular values does *A* have?



## REVIEW - CHAPTER 3

- Express the Euclidean condition number of a matrix in terms of its singular values.
- If *A* is a $2n \times n$ matrix, rank the following methods according to the amount of work required to solve the linear least squares problem $Ax \approx b$.
    - (a) QR factorization by Householder transformations
    - (b) Normal equations
    - (c) Singular value decomposition.
- List at least two applications for the singular value decomposition (SVD) of a matrix other then solving least squares problems.