

REVIEW - CHAPTER 1

- What three properties characterize a well-posed problem?
- List three sources of error in scientific computation.
- Explain the distinction between:
 - (a) truncation (or discretization) and rounding,
 - (b) absolute error and relative error,
 - (c) computational error and propagated data error,
 - (d) precision and accuracy,
 - (e) forward and backward error,
 - (f) how are forward and backward error related to each other?

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- (a) What is meant by the *conditioning* of a problem?
Is it affected by:
 - (b) the algorithm used to solve the problem?
 - (c) the precision of the arithmetic used to solve the problem?
- (a) Give an example to show that floating-point addition is not necessarily associative.
 - (b) Give an example to show that floating-point multiplication is not necessarily associative.

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- (a) In what circumstances does cancellation occur in a floating-point system.
- (b) Does it imply that the true result of the specific operation causing it is not exactly representable in the floating-point system?
- (c) Why is cancellation usually bad?
- Explain why an alternating infinite series, such as: $e^x = 1 + x + x^2/2! + x^3/3! + \dots$ for $x < 0$, is difficult to evaluate accurately in floating-point arithmetic.

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- (a) Explain the difference between the unit roundoff (ϵ), and the underflow level, UFL. Which one of these two quantities,
 - (b) depends only on the number of digits in the mantissa field?
 - (c) depends only on the number of digits in the exponent field?
 - (d) does *not* depend on the rounding rule used?
 - (e) is *not* affected by allowing subnormal numbers?

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- (a) Explain why a divergent infinite series, such as: $\sum (1/n)$, $n=1.. \infty$ can have a finite sum in floating-point arithmetic.
- (b) At what point will the partial sums stop to change?

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- Assume a decimal (base 10) floating-point system having machine precision $\epsilon = 10^{-5}$ and an exponent range of ± 20 . What is the result of each of the following floating-point arithmetic operations?
 - (a) $1 + 10^{-7}$
 - (b) $1 + 10^3$
 - (c) $1 + 10^7$
 - (d) $10^{10} + 10^3$
 - (e) $10^{10}/10^{-15}$
 - (f) $10^{-10} \times 10^{-15}$

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- Give four equivalent conditions for the non-singularity of an $n \times n$ matrix A .
- How many solutions can have a linear system $Ax = b$?
- (a) State one defining property of a *singular* matrix A .
- (b) Suppose that the linear system $Ax = b$ has two distinct solutions x and y . Use the property you gave in part a to prove that A must be singular.

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- Give the definition of the vector p -norm and particularly for $p=1, 2$ and ∞ .
- Give three properties of any vector p -norm.
- In 2-D, is it possible to have two vectors x and y that $\|x\|_1 > \|y\|_1$, but $\|x\|_\infty < \|y\|_\infty$? Give an example.
- Given a norm of vector x , give the definition of the matrix norm for an $n \times n$ matrix A , and particularly for $p=1$ and ∞ .
- Give five properties of any matrix norm.
- True or false: (a) The norm of a singular matrix is zero; (b) if $\|A\| = 0$, then $A = 0$; (c) $\|A\|_1 = \|A^T\|_\infty$.

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- (a) How is the condition number of a matrix A defined for a given matrix norm?
- (b) True or false: If A is any $n \times n$ nonsingular matrix, then $\text{cond}(A) = \text{cond}(A^{-1})$.
- (c) Write the relation between the relative change in the solution, and the relative change in the problem data and the condition number.
- (d) How is the condition number used in estimating the accuracy (the number of accurate digits) of a computed solution to a linear system $Ax = b$?

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- (a) In solving a linear system $Ax = b$, what is meant by the residual of an approximate solution x ?
- (b) How the relative change in the solution is bounded by the relative residual and the condition number of A ?
- (c) Does a small relative residual always imply that the solution is accurate? Why?
- (d) Does a large relative residual always imply that the solution is inaccurate? Why?

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