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Scientific Computing - Homeworks

7.5

a) Determine the polynom interpolant to the data

t	**1**	**2**	**3**	**4**
y	**11**	**29**	**65**	**125**

using the monomial basis .

The polynom has the form : $p(t) = x_1 + x_2 t + x_3 t^2 + x_4 t^3$.

Vandermonde matrix $A = [\ 1 , t_1 , t_1^2 , t_1^3\ ;\ 1 , t_2 , t_2^2 , t_2^3\ ;\ 1 , t_3 , t_3^2 , t_3^3\ ;\ 1 , t_4 , t_4^2 , t_4^3\ ]$.

$A x = y \Rightarrow$
$[\ 1, t_1, t_1^2, t_1^3\ ;\ 1, t_2, t_2^2, t_2^3\ ;\ 1, t_3, t_3^2, t_3^3\ ;\ 1, t_4, t_4^2, t_4^3\ ]\ [x_1 ; x_2 ; x_3 ; x_4] = [y_1 ; y_2 ; y_3 ; y_4]$
$\Rightarrow$
$[1, 1, 1, 1; 1, 2, 4, 8; 1, 3, 9, 27; 1, 4, 16, 64]\ [x_1 ; x_2 ; x_3 ; x_4] = [11; 29; 65; 125]$

we get the system :

- $x_1 + x_2 + x_3 + x_4 = 11$
- $x_1 + 2x_2 + 4x_3 + 8x_4 = 29$
- $x_1 + 3x_2 + 9x_3 + 27x_4 = 65$
- $x_1 + 4x_2 + 16x_3 + 64x_4 = 125$

$\Rightarrow$
$x_1 = 5; x_2 = 2; x_3 = 3 ; x_4 = 1 ; \Rightarrow x = [\ 5 ; 2 ; 3 ; 1\ ]$.

The interpolating polynomial is $p(t) = 5 + 2t + 3t^2 + t^3$.

b) Determine the Lagrange polynom interpolant to the resulting polynomial is equivalent to that obtained in part a .

The Lagrange polynomial of degree three is defined as :

$p(t) = y_1 l_1(t) + y_2 l_2(t) + y_3 l_3(t) + y_4 l_4(t)$

$= y_1 (t-t_2)(t-t_3)(t-t_4) / ((t_1-t_2)(t_1-t_3)(t_1-t_4)) + y_2 (t-t_1)(t-t_3)(t-t_4) / ((t_2-t_1)(t_2-t_3)(t_2-t_4)) +$
$y_3 (t-t_1)(t-t_2)(t-t_4) / ((t_3-t_1)(t_3-t_2)(t_3-t_4)) + y_4 (t-t_1)(t-t_2)(t-t_3) / ((t_4-t_1)(t_4-t_2)(t_4-t_3))$

$= 11 (t-2) (t-3) (t-4) / ((-1)(-2)(-3)) + 29 (t-1)(t-3)(t-4) / (1(-1)(-2)) +$
$65 (t-1)(t-2)(t-4) / (2 (1) (-1)) + 125 (t-1)(t-2)(t-3) / (3 (2) (1))$

$= 5 + 2t + 3t^2 + t^3$.