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Date: 18.01.2003
Homework number: 5
Homework Title: Exercise 5.3

Problem description:

Newton's method is sometimes used to implement the built-in square root function on a computer, with the initial guess supplied by a lookup table.

(a) What is the Newton iteration for computing the square root of a positive number y (i.e., for solving the equation $f(x) = x^2 - y = 0$, given y)?

(b) If we assume the starting guess has an accuracy of 4 bits, how many iterations would be necessary to attain 24-bit accuracy? 53-bit accuracy?

Problem solution:

(a) Use $x_{k+1} = x_k - f(x_k)/f'(x_k)$ to calculate given case.

(b) Newton's method has a convergence rate of 2 (quadratic), which doubles the number of accurate digits every iteration.

Results:

(a) Iteration for this case:

$$x_{k+1} = x_k - \frac{x_k^2 - y}{2x_k}$$

(b) To get 24-bit accuracy three iterations are needed (first iteration yields 8 accurate bits, second 16 and third 32). To get 53-bit accuracy four iterations are needed (fourth iteration yields 64 accurate bits).

Discussion and Comments: