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Scientific Computing - Homeworks

5.12 Newton's method for solving a scalar nonlinear equation *f*(x) = 0 requires computation of the derivative of *f* at each iteration . Suppose that we instead replace the true derivative with a constant value d, that is, we use the iteration scheme :

$$x_{k+1} = x_k - f(x_k) / d$$

a) under what condition on the value of d will this scheme be locally convergent ?

$g(x) = x - f(x) / d$

$g'(x) = 1 - f'(x) / d$

The condition for convergence is : $| g'(x^*) | < 1 \Leftrightarrow | 1 - f'(x^*) / d | < 1$

Case 1. $f'(x^*) / d <= 1 \Rightarrow | 1 - f'(x^*) / d | = 1 - f'(x^*) / d$ and : $1 - f'(x^*) / d < 1$
$\Rightarrow f'(x^*) / d > 0$
Case 2. $f'(x^*) / d > 1 \Rightarrow | 1 - f'(x^*) / d | = f'(x^*) / d - 1$ and : $f'(x^*) / d - 1 < 1$
$\Rightarrow f'(x^*) / d < 2$
From Case 1 and Case 2 :
$0 < f'(x^*) / d < 2 \Leftrightarrow$ d and $f'(x^*)$ must have the same sign , and
if $d > 0$, $f'(x^*) > 0$: $\Rightarrow 2d > f'(x^*)$
if $d < 0$, $f'(x^*) > 0$: $\Rightarrow 2d < f'(x^*)$

b) what will be the convergence rate, in general ?

$e_{k+1} = g(x_k) - g(x^*) = g(x_k) - g(x_k) + g'(x_k)(x_k - x^*) + g''(x_k)(x_k - x^*)^2 / 2$

$= g'(x_k)(x_k - x^*) + g''(x_k)(x_k - x^*)^2 / 2$

$= (1 - f'(x_k) / d) e_k + (f''(x_k) / 2d) e_k^2$

=> the convergence rate is linear , r = 1 .

c) is there any value for d that would still yield quadratic convergence ?

For quadratic convergence : $g'(x_k) = 0 \Rightarrow 1 - f'(x_k) / d = 0 \Rightarrow d = f'(x_k)$.

A constant value of d does not exist , d must be a function of f '(x) ,for a quadratic convergence .